Odd Vertex Magic Labeling

C.Y.Ponnappan [1], CT.Nagaraj[2], G.Prabakaran[3]

[1] Research Supervisor and Assistant Professor, Department of Mathematics , Government Arts College, Paramakudi-623 707 , Tamilnadu, India

[2] Assistant Professor, Department of Mathematics , Arignar Anna Arts & Science College, Krishnagiri – 635 001,Tamilnadu,India

[3] Assistant Professor, Department of Mathematics, Thiagarajar College, Madurai – 625 009, Tamilnadu, India

Abstract - A vertex-magic labeling is an assignment of the integers from 1,2,3,…….m+n to the vertices and edges of G so that at each vertex , the vertex label and the labels on the edges incident at that vertex add to a fixed constant. In this paper, we introduce a new concept odd s vertex labeling of a graph an f establish some families of a graphs have odd super vertex magic labeling

1. Introduction

In this paper, we consider only finite simple undirected graph. The graph G has vertex set V=V (G) and edge set E=E (G) and we take m=|E| and n =|V|. The set of vertices adjacent to a vertex u of G is denoted by N(u).

In the notion of a vertex-magic total labeling was introduced. This is an assignment of the integers from 1 to m+n to the vertices and edges of G so that at each vertex, the vertex label and the labels on the edges incident at that vertex, add to a fixed constant more formally, a one to one map f from V ∪ E onto the integers {1,2,3…..m+n} is a vertex-magic total labeling if there is a constant k and so that for every vertex u f(u)+∑f(uv) = k where the sum runs over all vertices v adjacent to u .

Definition: 1.1

A vertex magic labeling f is called odd vertex magic-labeling if

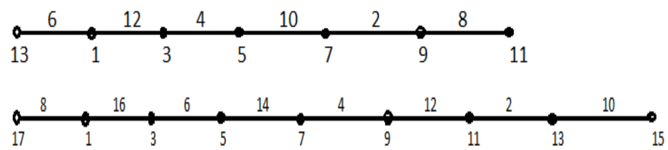
f : V → {1,3,5,……..2n-1} and f : E → {1,2,3,4……m+n} - {1,3,5,……..2n-1}

(if m ≥ n-1)

Otherwise :

f : E → {2,4,6,……..2m} and f : V → {1,2,3,4……m+n} - {2,4,6,……..2m}

Example:



2. Main Results

Theorem : 2.1

Let G be a nontrivial graph G with m ≥ n-1 is odd vertex magic labeling then the magic constant k is given by k = 3n-2 when m = n-1 otherwise $k = 1 + 2m + \frac{m^2}{n}+\frac{m}{n}$

Proof:

Let f be a odd vertex magic labeling of a graph G with the magic number k.

Then f(v)={1,3,5……….2n-1} and $k=f(u)+\sum_{v\in N(u)} f(uv)$ ∀ u∈V.

Case(i) m = n-1

n+m = 2n - 1

f : V→{1,3,5,7,……2n-1}

f : E→{2,4,6…………..2n-2}

$nk = n^2+2(2+4+6+\ldots\ldots 2n-2)$

$nk = n^2+2n(n-1)$

k = 3n-2

case(ii) m>n-1

f : V→{1,3,5,7,……2n-1}

f:E→{2,4,6,8…………2n-2,2n,2n+1,2n+2………n+m}

f:E→{2,4,6,8…………2n-2,2n,2n+1,2n+2………2n+(m-n)}

$nk = n^2+2(2+4+6+\ldots\ldots 2n-2+2n)+2(2n+1+2n+2+\ldots\ldots+2n+(m-n))$

nk = n^2+2n(n+1)+4n(m-n)+(m-n)(m-n+1)

k = 1+2m+ $\frac{m^2}{n}$+$\frac{m}{n}$

Theorem : 2.2

Let G be a nontrivial graph G with m <n-1 is odd vertex magic labeling then the magic constant k is given by

$$K = \frac{n}{2}+m+\frac{1}{2}+\frac{3}{2n}(m^2+m)$$

Proof

Let f be a odd vertex magic labeling of a graph G with the magic number k

Let G be a graph with m<n-1

Then k =f(u)+$\sum_{v\in N(u)} f(uv)$ ∀ u∈V.

f:E→{2,4,6………2m} and f:V→{1,2,3………m+n}-{2,4,6………….2m}

nk = 1+2+3+……….+n+m)-(2+4+6+………+2m)+2(2+4+6+………..+2m)

nk =$\frac{n^2}{2}$ +nm+$\frac{n}{2}$+3$\frac{m^2}{2}$+3$\frac{m}{2}$ /

k =$\frac{n}{2}$+m+$\frac{1}{2}$+$\frac{3}{2n}(m^2+m)$

Theorem:2.3

A path P_n is odd super vertex magic labeling if and only if n is odd and n≥ 3

Proof:

Suppose there exists an odd vertex magic labeling f of P_n with the magic number k.

Then by theorem 2.1

K = 3n-2

To prove that n is odd:

Suppose n is even. Then k = 3n-2 is even. For any odd vertex magic labeling f, f(u)+∑f(uv)=k ∀ u∈V. In particular if u is a pendent vertex of P_n then f(u)+f(uv) =k ,which is a contradiction. Since f(u) is odd and f(uv) is even.

Therefore n is odd.

Converse :

Let n be an odd integer and n ≥3

V(P_n)={v_1,v_2,v_3………..v_n} and E(P_n)={e_i=v_iv_{i+1}/1≤i≤n-1}

Define f:V∪E→{1,2,3……………2n-1} as follows

f(v_1) = 2n-1

f(v_i) = 2i-3 , 2≤i≤n

$$f(e_i) = \begin{cases} n-i & if\ i\ is\ odd \\ 2n-i & if\ is\ even \end{cases}$$

It is easily seen that f is an odd vertex magic labeling with the magic number 3n-2.

Theorem : 2.4

A cycle C_n is odd vertex magic labeling iff n is odd.

Proof:

Suppose there exists an odd vertex magic labeling f of C_n with the magic number k

Then by theorem 2.1

K = 1+2n+ $\frac{n^2}{n}$ +$\frac{n}{n}$

K = 3n+2

To prove that n is odd

Suppose n is even. Then k=3n+2 is even. For any odd vertex magic labeling f, f(u)+∑f(uv)=k ∀ u∈V. In particular f(v_i)+f(v_i v_{i-1})+f(v_i v_{i+1})=k, which is a contradiction. Since f(v_i) is odd and f(v_i v_{i-1}) and f(v_i v_{i+1}) are even. Therefore n is odd.

Converse:

Let n be an odd integer .

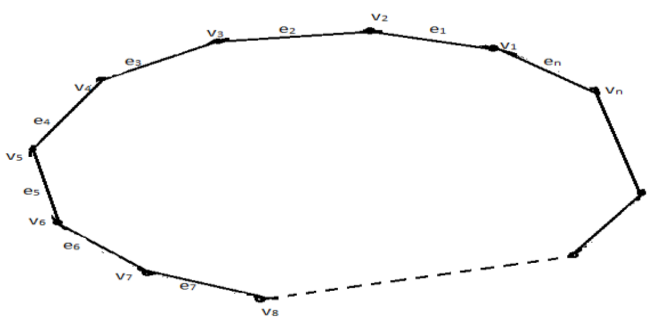
V(C_n) = {v_1,v_2,v_3………..v_n} and E(C_n)={e_i=v_iv_{i+1}/1≤i≤n-1}∪{e_n=v_nv_1}

Define f:V∪E→{1,2,3…………..2n} as follows

f(v_i)=2i-1 1≤i≤n

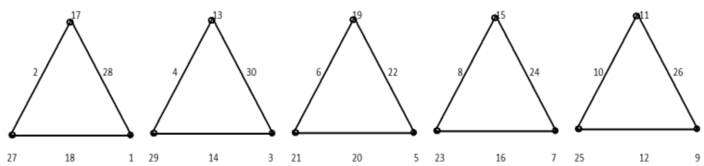
$$f(e_i) = \begin{cases} 2n-i+1 & if\ i\ is\ odd \\ n-i+1 & if\ i\ is\ even \end{cases}$$

It is easily seen that f is an odd vertex magic labeling with the magic number 3n+2.



3. ODD VERTEX MAGIC LABELING ON A DISCONNECTED GRAPH

In this section, we give an odd vertex magic labeling for the disconnected graph m C_3 that is ,the disjoint union of m cycles of length 3, where m is odd.



Theorem : 3.1

m C_3 is odd vertex magic labeling if and only if m is odd.

Proof:

Suppose there exists an odd vertex magic labeling of m C_3 with the magic number k.Then by theorem 2.1

$K = 1+6m+\frac{(3m)^2}{3m}+\frac{3m}{3m}$

$K = 1+6m+3m+1$

$K = 9m+2$

To prove that m is odd

Suppose m is even. Then k=9m+2 is even. For any odd vertex magic labeling f, $f(u)+\sum f(uv)=k$ $\forall u \in V$. In particular $f(v_i)+f(v_i v_{i-1})+f(v_i v_{i+1})=k$, which is a contradiction. Since $f(v_i)$ is odd and $f(v_i v_{i-1})$ and $f(v_i v_{i+1})$ are even. Therefore m is odd.

Let m be odd integer. Assume that the graph m C_3 has vertex set $V=V_1\cup V_2\cup \ldots\ldots\cup V_m$ where $V_i = \{v_{i1},v_{i2},v_{i3}\}$ and the edge $E = E_1\cup E_2\cup\ldots\ldots\cup E_m$ where $E_i = \{e_{i1},e_{i2},e_{i3}\}$ and $e_{ij} = v_{ij}v_{i(j+1)}$ for $1\le i\le m$, $1\le j\le 2$, $e_{i3} = v_{i3}v_{i1}$

Define $f;V\rightarrow\{1,3,5\ldots\ldots 6m-1\}$ as follows

$f(v_{i1}) = 2i-1$, $I = 1,2\ldots\ldots m$

$$f(v_{i2}) = \begin{cases} 5m+2i & 1\le i\le (m-1)/2 \\ 3m+2i & \frac{m+1}{2}\le i\le m \end{cases}$$

$$f(v_{i3}) = \begin{cases} 4m-4i+1 & 1\le i\le (m-1)/2 \\ 6m-4i+1 & \frac{m+1}{2}\le i\le m \end{cases}$$

Define $f:E\rightarrow\{2,4,6\ldots\ldots 6m\}$ as follows

$$f(e_{i1}) = \begin{cases} 4m-4i+2 & 1\le i\le (m-1)/2 \\ 6m-4i+2 & \frac{m+1}{2}\le i\le m \end{cases}$$

$f(e_{i2})=2i$, $i=1,2\ldots\ldots m$

$$f(e_{i3}) = \begin{cases} 5m+2i+1 & 1\le i\le (m-1)/2 \\ 3m+2i+1 & \frac{m+1}{2}\le i\le m \end{cases}$$

It is easily verified that f is an odd vertex labeling of m C_3 with k=9m+2

4. SUNS

An n-sun is a cycle C_n with an edge terminating in a vertex of degree 1 attached to each vertex.

Theorem : 4.1

All n-suns are not odd vertex magic labeling

Proof :

All suns 2n vertices and 2n edges.

For any odd vertex magic labeling **f,** $f(u)+\sum f(uv)=k$ $\forall u \in V$. f(u) is odd and each f(uv) is even. Therefore k is odd.

Then by theorem 2.1 k = 1+4n+2n+1 = 6n+2 is even for any n

Whish is a contradiction to k is odd.

Therefore all n-suns are not odd vertex magic labeling

5. KITE

An (n,t)-kite consists of a cycle of length n with a t-edge path (the tail) attached to one vertex.

Theorem : 5.1

An kite (3,t) is an odd vertex magic labeling iff t is even

Proof:

Assume that kite (3,t) is an odd super vertex magic labeling

n = t+3

m = t+3

Then by theorem 2.1 k = 1+(2t+6)+(t+3) +1 =3 t+11

To prove that t is even

Suppose that t is odd

K = 3t+11 is even. In particular if u is a pendent vertex of kite (3,t) ,then f(u)+f(uv) = k ,which is a contradiction. Since f(u) is odd and f(uv) is even.

Therefore t is even

Converse Assume that t is even

$f(v_i) = 2i-1, \quad 1 \le i \le t+3$

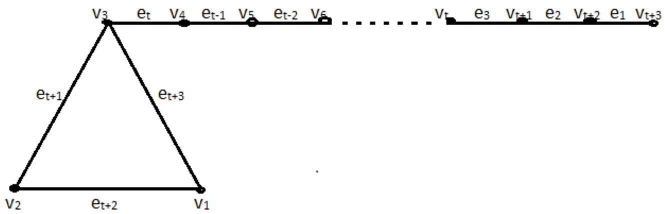
$f(e_i) = i$ if i is even, $i \le t$

$f(e_{t+2}) = 2t+6$

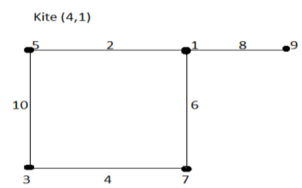
$f(e_i) = i+t+5$, if i is odd, $i \le t-1$

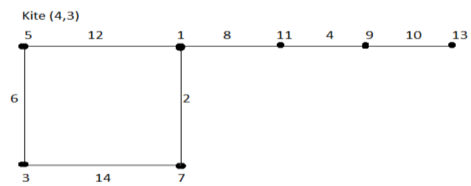
$f(e_{t+1}) = t+2$

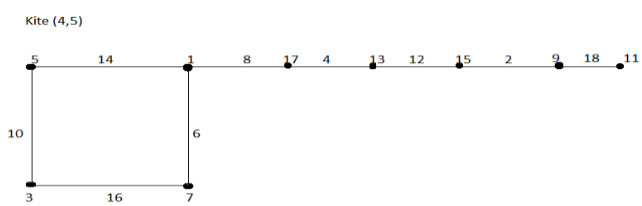
$f(e_{t+3}) = t+4$



Example :







References

1. V. Swaminathan P. Jeyanthi Super vertex magic labeling Indian j.pure appl .math.,34(6) 935-939,June 2003.

2. W D. Wallis ,Magic Graphs,Birkhauser,Basel,2013

3. Joseph A. Gallian, A Dynamic survey of Graph Labeling, The Electronic Journal of Combinatories.