Meanlives Predictions for Rotational Excited Ground Band States for Even-Even Nuclei in Rare Earth and Actinide Series

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Abstract

The measurement of mean life of excited states in nuclei is one of the most active areas of nuclear structure physics. In this work the asymmetric rotor model of Davydov- Filippov (DF) has been employed to study the mean lifetime in the rotational excited ground state band even-even nuclei of rare earth and actinide series which comprises of 57 nuclides. The mean life for E2-transitions raging up to 12^+ spin states transitions have been studied in detailed in the spectra of nuclei whose mass number ranges as 150<=A<=190 and A>=228, and for those the first excited state 2^+ and the second excited state $2^{+'}$ gamma band energies are available. The best input parameters have been employed. These input parameters includes the energy involved in the transition, the total internal conversion coefficient, and the empirical reduced transition probability B(E2). The mean lifetime have been calculated by using the most recent available experimental data. Comparison of results of theoretical calculations with the corresponding experimental data shows a very good agreement, including high angular momentum states. This work has incorporated many nuclides and transitions for which neither experimental nor theoretical values are available.

Keywords: *asymmetric rotor model, reduced transition probabilities, internal conversion coefficient, mean life.*

I. INTRODUCTION

An interesting feature of nuclear structure is nuclear deformation. It has been known for a long time, that heavy nuclei with many valence particles of both kinds tend to take on a static prolate axially symmetric quadrupole deformation in their ground state. Regular rotational excitation bands are beautiful evidence of this fact. Excitations and behavior of deformed even-even nuclei can be very successfully described using nuclear collective models (Bohr & Mottelson, 1998).

Further from closed shells, the accumulating p-n interaction strength leads to additional configuration mixing and deviations from spherical symmetry even in the ground state, and so we now turn to consider nuclei with stable and permanent deformations. The lowest applicable shape component is a quadrupole distortion. There can also be octupole and hexadecapole shapes in the nuclear deformation.

Nuclear shape is usually specified in terms of the two nuclear deformation parameters β and γ. The β parameter represents the extent of quadrupole deformation, while γ gives the degree of axial asymmetry. Nuclear triaxiality is associated with the breaking of axial symmetry of the quadrupole deformation (Nadirbekov, 2016). In the frame work of the rigid tri-axial rotor model, deformation parameters β and γ are extraxted from both level energies and E2 transition rates in even-even nuclei. The rigid tri-axial rotor model considers the nucleus as a rigid rotor with rigid tri-axial asymmetry as specified by β and γ (Varshney, 1982).

The nuclear deformation parameters β and γ of the collective model are basic description of the nuclear equilibrium shape and structure, while values for these variables have been discussed for many nuclei (Singh, 2011). It has been shown by the study of even-even nuclei in the regions A <=150 and A >=190 that the properties of their excited levels can be accounted for by considering the rotational motion of non-axial (non-axially symmetric) nuclei or nuclear vibrations.

A new collective theory of the behavior of nuclei has been developed by Davydov and Filippov (DF) taking into account possible violations of the axial symmetry of the nucleus. This violation affects the rotational spectrum of the axial even nucleus, and some new rotational states with total angular momenta of 2, 3, 4,… appear. A deformed nucleus has a rotational degree of freedom. For even-even nuclei, the 0^+ state is always the ground state. The next states with a rotational degree of freedom are $I^\pi = 2^+, 4^+, 6^+, 8^+,\ldots$ on symmetry grounds (Pearson, 2008).

Even-even nuclei are known to have 0^+ ground states and several low-energy integer spin states. The transition strengths between these levels are sufficiently strong and well established to support the view that most nuclei are collective. Davydov and Filippov axially asymmetric model has been found quite suitable

(Allmond, 2007; Davydov, 1958; Varshney, 2009) in explaining the rotational levels of the deformed even-even nuclei, the large electric quadrupole moments and the transition probabilities.

In this study the systematic study of the properties of nuclear rotational excited levels in the deformed even-even nuclei will be considered in the framework of Asymmetric Rotor Model of Davydov and Filippov. The aim of the present work is to apply the Davydov-Fillipov model for calculating the values of the mean-lives for the rotational excited ground band states even-even nuclei of rare earth and actinide series.

The measurement of the mean lifetimes of rotational states provides the means to probe the nuclear shape at intermediate to high spin, since the electric quadrupole transition matrix elements in a rotational band are simply related to the magnitude of the quadrupole deformation of the intrinsic nuclear state. (Smith, 2012).

II. METHODS

The experimental transition probabilities in units of $e^2.b^2$ used in this study is computed using (Biniyam & Chaubey, 2017)

$$B(E2{:}I+2\rightarrow I)_{exp} = \frac{0.0566}{E_\gamma{}^5(1+\alpha_T)T_{1/2}} \qquad (2.1)$$

where Eγ is the energy involved in the transition and is expressed in MeV, T1/2 is the half-life of the excited state in Ps, and α_T are the total internal conversion coefficients calculated using Bricc online software (Australian National University, 2011).

The experimental transition probabilities in units of $e^2.b^2$ is related to the experimental mean-life by the expression (Raman, Nestor & Tikkaneny, 2001; Varshney, 1982)

$$B(E2{:}I+2\rightarrow I)_{exp} = \frac{0.08162}{E_\gamma{}^5(1+\alpha_T)\tau} \qquad (2.2)$$

Where τ is the mean- life of the excited state in Ps.

Using equation (2.2), we can rewrite the experimental mean-life as

$$\tau = \frac{0.08162}{E_\gamma{}^5(1+\alpha_T)B(E2{:}I+2\rightarrow I)exp} \qquad (2.3)$$

The electric quadrupole transition probabilities for transitions inside the ground rotational band between two states of spin I and I' are described by the following empirical formula (Varshney, 1982):

$$B(E2{:}I\rightarrow I')_{emp} = \frac{5e^2Q_0^2}{32\Pi}(2I00|I'0)^2\left\{1+\frac{3-2\sin^2(3\gamma)}{[9-8\sin^2(3\gamma)]^{1/2}}\right\} \qquad (2.4)$$

Where $(2I00|I'0)$ are Clebsch-Gordon coefficients in the notation (2Jmm'|J'm').

For transition between two states of ground rotational band, the Clebsch-Gordon coefficients have the form $(2I0|I'0)^2 = \frac{3}{2}\frac{(I+1)(I+2)}{(2I+3)(2I+5)}$ in decay or de-excitation from I+2→I. Hence

$$B(E2{:}I+2\rightarrow I)_{emp} = \frac{15e^2Q_0^2}{64\Pi}\frac{(I+1)(I+2)}{(2I+3)(2I+5)}\left\{1+\frac{3-2\sin^2(3\gamma)}{[9-8\sin^2(3\gamma)]^{1/2}}\right\} \qquad (2.5)$$

Where Qo is the intrinsic quadrupole moment given by (Raman, S., Nestor, C. W., J. R., & Tikkaneny, P., 2001)

$$Q_o = \left[\frac{16\Pi}{5}\frac{B(E2)\uparrow}{e^2}\right]^{1/2} \qquad (2.6)$$

Where B(E2)↑is the reduced electric quadrupole moment transition probability between the 0^+ ground state and the first 2^+ state in even-even nuclides. The B(E2)↑values are basic experimental quantities that do not depend on nuclear models.

In this research we have employed the most widely used method to calculate the asymmetry parameter γ which is used in empirical calculation. The asymmetry parameter is evaluated from the ratio of two band head energies $E2^{+'}/E2^+$ (Singh et al, 2013; Varshney et al, 2011),

where $E2^{+'} = \frac{9}{\sin^2(3\gamma)}\left[1+\sqrt{1-\frac{8\sin^2(3\gamma)}{9}}\right]\frac{\hbar^2}{4B\beta^2}$ & $E2^{+} = \frac{9}{\sin^2(3\gamma)}\left[1-\sqrt{1-\frac{8\sin^2(3\gamma)}{9}}\right]\frac{\hbar^2}{4B\beta^2}$ (2.7)

So that we can write for the asymmetric parameter as

$$\gamma = \frac{1}{3}\sin^{-1}\left\{\frac{9}{8}\left[1-\frac{(m-1)^2}{(m+1)^2}\right]\right\}^{1/2} \qquad (2.8)$$

Where m is the energy ratio $E2^{+'}/E2^+$.

When the ratio of experimental B(E2) to empirical B(E2) values versus γ-values are very close to unity the empirical B(E2) values may be used (Varshney, 1982) for the prediction of meanlives for those transitions where neither the experimental B(E2) values nor the meanlives are known. So the experimental meanlife can be rewritten replacing by empirical reduced transition probability as

$$\tau = \frac{0.08162}{E_\gamma{}^5(1+\alpha_T)B(E2{:}I+2\rightarrow I)emp} \qquad (2.9)$$

This is an empirical equation used in this study to predict the meanlives of the transitions.

III. RESULT

In this section the energy ratio and B(E2) values ratio are figured, and the empirical meanlives computed have been tabulated. Table 1 gives meanlives for transition between rotational states of 0^+, 2^+, 4^+, 6^+, 8^+, 10^+ and 12^+.

In this study we have selected nuclides which are rotational nuclei. The rotational states are determined by the energy ratio E_4/E_2, as shown in the Figure 1.

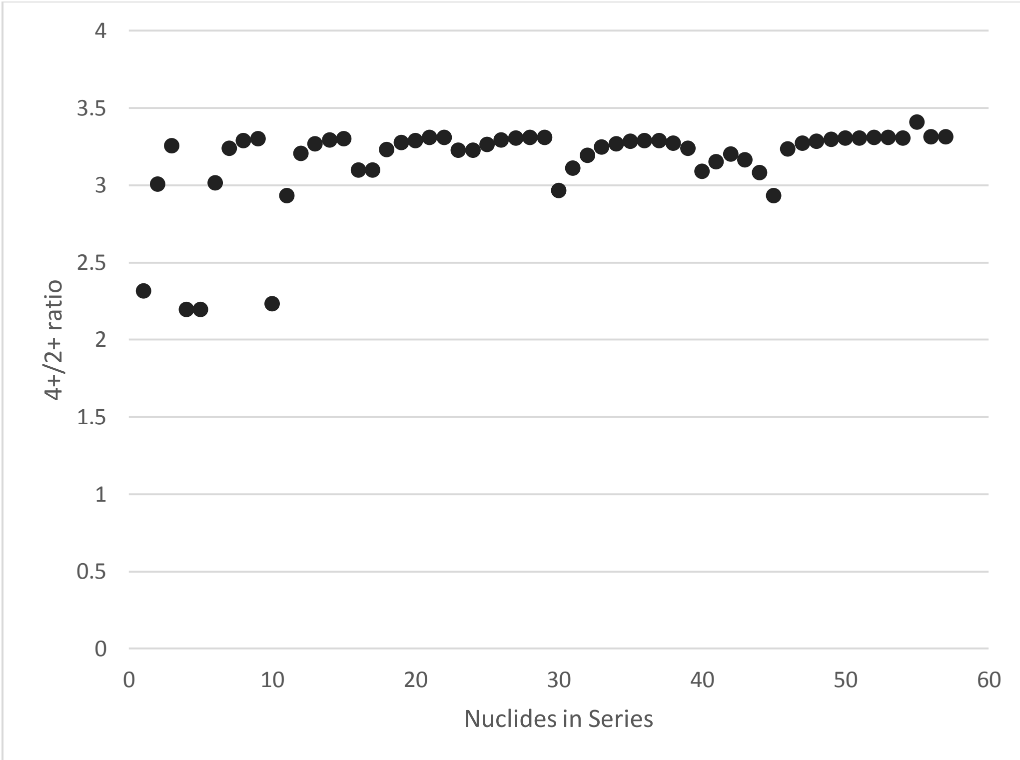
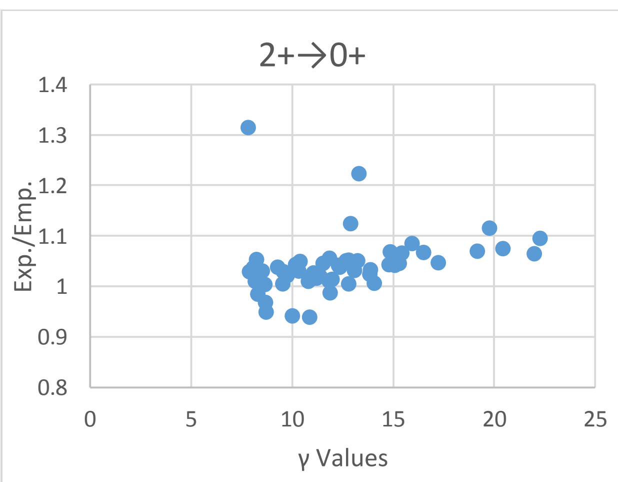
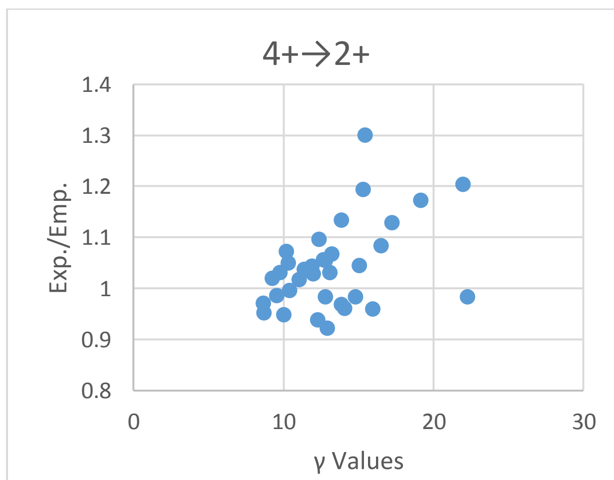


Fig. 1. Energy Ratio E_4/E_2 for Nuclides.

The ratio of experimental B(E2) to empirical B(E2) values versus γ-values are shown in the figure 2. It can be seen that the ratio is in most cases very close to unity.





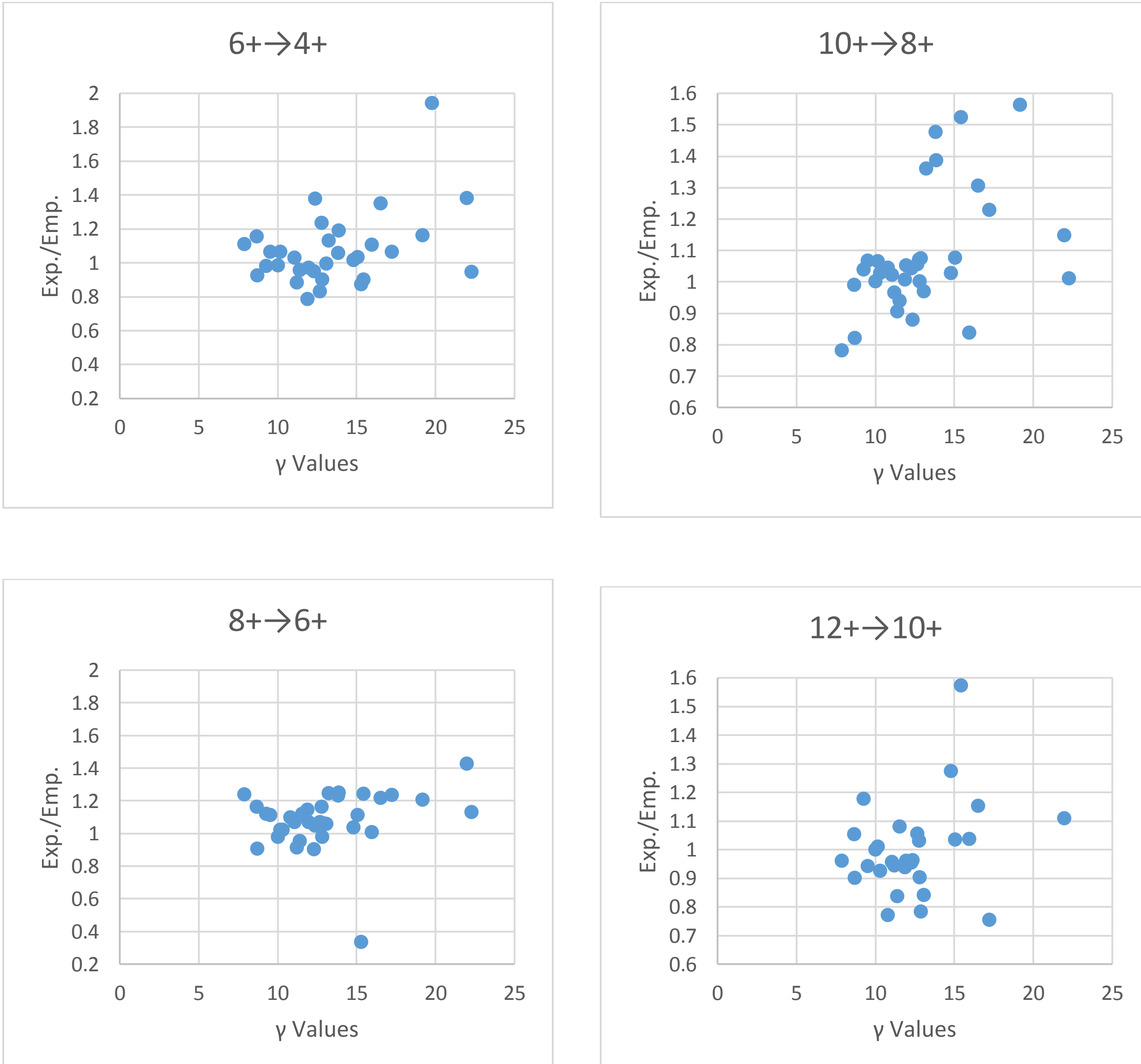


Fig. 2. Ratio of Experimental/Empirical B(E2) Values Versus γ-Values.

Table 1: Experimental and Empirical Meanlives of Transitions

Nucle-us	Theoretical Mean lifetime						Experimental Mean lifetime					
	2+→0+	4+→2+	6+→4+	8+→6+	10+→8+	12+→10+	2+→0+	4+→2+	6+→4+	8+→6+	10+→8+	12+→10+
$^{150}_{62}$Sm	75.018	13.596	6.1502	3.595	2.524	2.118	69.826	9.3775	2.0198	-	-	-
$^{152}_{62}$Sm	2121.137	88.76	16.503	5.663	2.709	1.553	2019.77	83.244	14.571	4.545	1.991	-
$^{154}_{62}$Sm	4374.107	244.555	34.857	9.478	3.775	1.891	4356.94	248.144	32.749	8.512	3.535	2.005
$^{152}_{64}$Gd	52.179	15.287	7.011	4.164	2.952	2.234	46.743	10.532	3.607	-	-	-
$^{154}_{64}$Gd	1768.477	73.974	13.411	4.613	2.201	1.281	1711.04	65.210	11.253	3.693	1.587	-
$^{156}_{64}$Gd	3270.808	164.103	23.525	6.675	2.805	1.52	3188.36	161.438	22.795	6.232	2.741	1.587
$^{158}_{64}$Gd	3744.24	223.806	29.879	7.529	2.747	1.308	3635.59	213.519	23.083	7.358	2.669	1.414
$^{160}_{64}$Gd	3936.074	262.891	34.97	8.529	3.054	1.39	3880.85	-	-	-	-	-
$^{154}_{66}$Dy	41.607	10.42	4.614	2.762	1.989	1.487	39.097	8.656	3.333	1.933	1.731	1.342
$^{156}_{66}$Dy	1263.482	55.49	10.81	3.945	1.956	1.202	1187.34	42.704	11.974	3.174	1.284	0.765
$^{158}_{66}$Dy	2501.129	109.385	16.235	4.878	2.164	1.264	2394.87	103.874	13.129	4.184	2.034	1.226
$^{160}_{66}$Dy	2944.904	154.837	21.098	5.685	2.269	1.206	2922.9	148.598	26.834	4.948	2.251	1.284
$^{162}_{66}$Dy	3219.089	195.714	25.813	6.477	2.386	1.124	3173.93	190.436	26.546	6.059	2.265	1.169
$^{164}_{66}$Dy	3549.189	271.66	37.317	9.396	3.445	1.626	3448.04	289.982	39.241	10.387	3.304	1.702
$^{160}_{68}$Er	1374.958	48.644	8.0722	2.734	1.351	0.866	1325.84	46.599	7.791	2.453	1.255	0.837
$^{162}_{68}$Er	2044.02	83.715	11.85	3.427	1.493	0.869	1687.95	-	-	-	-	-
$^{164}_{68}$Er	2352.488	114.168	15.147	3.973	1.55	0.78	2120.76	124.072	-	3.737	1.443	0.995
$^{166}_{68}$Er	2719.231	179.423	24.61	6.488	2.591	1.372	2625.7	170.238	29.604	6.059	2.453	1.298
$^{168}_{68}$Er	2773.946	179.916	23.07	5.441	1.881	0.861	2712.27	164.467	16.735	5.194	2.135	0.894
$^{170}_{68}$Er	2809.693	190.166	24.467	5.821	2.009	0.888	2728.14	203.997	-	5.194	2.135	0.822
$^{166}_{70}$Yb	1822.283	78.549	11.211	3.268	1.399	0.777	1788.94	76.3186	11.253	3.087	1.443	0.923
$^{168}_{70}$Yb	2217.06	133.698	18.775	5.277	2.238	1.25	2120.76	-	-	-	-	-
$^{170}_{70}$Yb	2301.982	144.485	18.842	4.711	1.751	0.856	2315.53	-	-	4.285	1.674	1.111
$^{172}_{70}$Yb	2430.613	178.817	23.547	5.666	1.987	0.884	2380.45	176.009	23.949	5.049	1.904	0.750
$^{174}_{70}$Yb	2540.607	201.103	26.634	6.392	2.288	1.002	2582.42	207.748	23.083	5.482	2.308	0.952
$^{176}_{70}$Yb	2624.454	170.946	21.575	5.188	1.858	0.867	2539.14	158.696	20.198	5.049	1.731	0.851
$^{166}_{72}$Hf	751.063	27.505	5.396	2.153	1.25	0.963	717.019	24.237	5.049	1.731	1.010	1.298
$^{168}_{72}$Hf	1329.222	51.011	8.663	2.992	1.499	0.955	1284	51.937	8.512	2.885	1.457	0.750
$^{170}_{72}$Hf	1720.229	87.829	14.068	4.519	2.199	1.315	1731.23	89.4471	15.581	4.617	2.193	1.457
$^{172}_{72}$Hf	2260.969	119.758	17.011	4.854	2.066	1.141	2236.18	-	-	-	-	-
$^{174}_{72}$Hf	2215.286	128.012	17.483	4.866	2.036	1.128	2394.87	-	-	-	-	-
$^{176}_{72}$Hf	2153.194	129.954	17.366	4.479	1.747	0.876	2063.05	-	-	-	-	-
$^{178}_{72}$Hf	2169.649	112.017	14.287	3.653	1.436	0.764	2135.19	-	16.158	3.996	1.486	0.808
$^{182}_{74}$W	2015.85	92.612	11.341	2.764	0.994	0.459	1975.05	89.447	11.830	2.899	1.096	0.548
$^{184}_{74}$W	1846.191	66.966	8.237	2.049	0.788	0.4	1804.81	69.249	7.791	1.659	0.534	-
$^{186}_{74}$W	1606.503	50.345	6.384	1.603	0.606	0.302	1494.63	52.514	5.771	1.587	0.721	0.291
$^{180}_{76}$Os	1194.089	46.486	8.465	3.35	2.007	1.475	1154.16	38.953	9.666	9.955	-	-
$^{182}_{76}$Os	1239.631	45.659	7.224	2.492	1.483	1.46	1172.91	-	-	-	-	-
$^{184}_{76}$Os	1699.2	63.703	8.894	2.5	1.026	0.54	1708.15	66.364	-	-	-	-
$^{186}_{76}$Os	1334.986	41.402	5.91	1.74	0.773	0.465	1262.36	38.231	4.371	1.428	0.592	0.404
$^{188}_{76}$Os	1088.369	32.784	5.204	1.69	0.857	0.665	1024.31	27.988	4.472	1.399	0.548	-
$^{190}_{76}$Os	570.99	20.011	3.558	1.242	0.686	-	523.698	20.342	3.751	1.096	0.678	-
$^{228}_{90}$Th	582.744	240.525	82.087	31.03	14.502	8.063	584.291	236.602	-	-	-	-
$^{230}_{90}$Th	536.14	238.547	84.478	30.766	13.57	7.03	510.714	239.487	-	-	-	-
$^{232}_{90}$Th	457.622	219.934	88.734	33.766	14.988	7.924	497.73	236.602	90.890	34.625	15.004	7.935
$^{234}_{92}$U	351.746	186.446	93.897	41.405	19.408	10.22	363.559	-	-	-	-	-
$^{236}_{92}$U	312.723	167.304	76.756	31.232	13.689	6.88	337.591	178.894	83.676	34.625	16.735	7.646
$^{238}_{92}$U	300.653	160.415	75.765	30.747	13.72	6.827	292.867	-	-	-	-	-

$^{238}_{94}$Pu	245.246	132.951	66.981	28.396	12.311	5.965	255.357	-	-	-	-	-
$^{240}_{94}$Pu	238.71	131.521	69.244	30.747	13.851	6.872	236.602	-	-	-	-	-
$^{242}_{94}$Pu	230.531	124.636	61.591	26.017	11.397	5.63	227.946	-	-	-	-	-
$^{244}_{94}$Pu	224.729	115.49	57.35	23.372	11.026	4.99	223.618	-	-	-	-	-
$^{244}_{96}$Cm	178.938	99.191	54.151	24.631	-	-	139.941	-	-	-	-	-
$^{246}_{96}$Cm	176.705	97.464	53.81	24.268	-	-	174.566	-	-	-	-	-
$^{248}_{96}$Cm	174.69	96.371	52.153	23.391	13.075	5.249	174.566	112.53	47.609	19.044	13.561	5.482
$^{250}_{98}$Cf	138.875	76.895	43.69	21.521	-	-	141.384	-	-	-	-	-
$^{252}_{98}$Cf	133.186	71.64	-	-	-	-	132.728	-	-	-	-	-

The experimental meanlives shown here are computed from halflives of the transition states from table of Isotopes (Firestone, 1999) using the known relation $\tau = \frac{T_{1/2}}{\ln 2}$.

IV. DISCUSSION

In this study the meanlives of rotational excited ground band states of even-even nuclei of rare earth and actinide series have been computed by making use of the empirical reduced transition probabilities. This is because, as can be seen from figure 2, the transition probability ratio is very close to unity.

As clearly seen from figure 1 the energy ratio in almost all nuclides under study fall beyond 2.5 where this is the rotational region. So that almost all the nuclides considered are rotational in nature where nuclear deformation is highly observed, and that the asymmetric rotor model assumed to work best.

The meanlives of transitions are dependent inversely upon the transition gamma energy to the power of five, the total internal conversion coefficient, and the empirical reduced transition probability (See Equation 2.9). The empirical reduced transition probability in turn is proportional to the square of the intrinsic quadrupole moment (Qo) (See Equation 2.5).

The intrinsic quadrupole moment in turn directly proportional to the elongation deformation parameter β. So in these empirical calculations both the asymmetric parameter γ and the elongation parameter β have been employed. Actually these parameters are so important in determining the actual nuclear deformation properties. The empirical reduced transition probabilities calculated are in a very good agreement (Biniyam & Chaubey, 2017) especially for lower transitions. There is a difficulty in determining the total ICC values for transition gamma energy greater than 400 KeV using the Bricc software. The transition gamma energy get increases for higher transitions and are observed to be greater than the mentioned value especially for the rare earth series.

We have found a very good agreement except for $^{150}_{62}$Sm & $^{152}_{64}$Gd at $4^+ \to 2^+$ and $6^+ \to 4^+$ transitions, $^{154}_{66}$Dy & $^{180}_{76}$Os at $8^+ \to 6^+$ transition, $^{156}_{66}$Dy, $^{184}_{74}$W & $^{188}_{76}$Os at $10^+ \to 8^+$ transition, and $^{156}_{66}$Dy at $12^+ \to 10^+$ transition. In these nuclei the percentage difference is found to be large, in most cases greater than 40%.

The percentage difference between experimental and empirical meanlives, in most cases, is below 10%. This percentage difference get increases as the level of transition get higher. This difference could be attributed to the fact that the total internal conversion coefficient could not be computed for the transition gamma energy greater than 400KeV. This may be the real cause for most cases since the large percentage difference is observed for those transitions where the transition gamma energy is beyond the mentioned value. And the percentage difference observed in actinide series is very small. This may be due to a smaller transition gamma energy than the minimum expected in the Bricc online software.

V. CONCLUSION

Life time predictions have been made for the 57 rotational excited states in the ground state band of rare earth and actinide series. We have compared our predictions of mealives with their respective experimental values, if known, and have found a very good agreement, and a good agreement at higher transitions for the rare earth series. In general the percentage difference increases as the transitions get higher.

So we conclude that the predicted meanlives in this study are observed to be in a very good agreement, especially at lower transitions, and that the asymmetric rotor model could be helpful in predicting the probable meanlives of rotational excited even-even ground state band.

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