Structural Design Optimization using Parameterized P-norm Based Geometric Programming Technique

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Abstract— *In this paper we will make an approach to solve single objective structural model using parameterized p-norm based fuzzy Geometric Programming technique. A structural design model in fuzzy environment has been developed. Here p-norm based generalised triangular fuzzy number (GTFN) is considered as fuzzy parameter so that the decision maker can take advantage of no-exact parameter. Generalised triangular p-norm is discussed with their basic properties and some special cases. In this structural model formulation, the objective function is the weight of the truss; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. A classical truss optimization example is presented here in to demonstrate the efficiency of our proposed optimization approach. The test problem includes a two-bar planar truss subjected to a single load condition. This approximation approach is used to solve this single-objective structural optimization model. The model is illustrated with numerical examples.*

Keywords— *Generalized Triangular Fuzzy Number, p-norm, Geometric Programming, Single Objective Optimization, Structural Optimization.*

I. Introduction

Optimization seeks to maximize the performance of a system, part or component, while satisfying design constraints. One common form of optimization is trial and error and is used every day. We make decisions, observe the result, and change future actions depending on the success of those decisions. When performing optimization, we wish to minimize (or maximize) the structural design, while considering both design variables and design constraints. Design variables are variables the designer or engineer can freely choose between, for example the thickness of a wall, the material chosen, and the width of a part. The resulting stress, deflection, volume, natural frequency and other typical performance measures are often considered either as objective functions or as constraints.

In practice, the problem of structural design may be formed as a typical non-linear programming problem with non-linear objective function and constraints functions in fuzzy environment. Zadeh [1] first introduced the concept of fuzzy set theory. Then Zimmermann [2] applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with several objective functions. Some researchers applied the fuzzy set theory to structural model. For example, Wang et al. [3] first applied -cut method to structural designs where the non-linear problems were solved with various design levels, and then a sequence of solutions were obtained by setting different level-cut value of Rao[4] applied the same α -cut method to design a four–bar mechanism for function generating problem. Structural optimization with fuzzy parameters was developed by Yeh et al. [5].Xu[6] used two-phase method for fuzzy optimization of structures. Shih et al. [7] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources. Shih et al.[8] developed an alternative -level-cuts methods for optimum structural design with fuzzy resources. Prabha et al.[18] presents an efficient algorithm to optimize fuzzy transportation problem.

Geometric Programming (GP) method is an effective method used to solve a non-linear programming problem like structural problem. It has certain advantages over the other optimization methods. Here, the advantage is that it is usually much simpler to work with the dual than the primal one. Solving a non-linear programming problem by GP method with degree of difficulty (DD) plays essential role. (It is defined as DD = total number of terms in objective function and constraints – total number of decision variables – 1). Since late 1960's, GP has been known and used in various fields (like OR, Engineering sciences etc.). Duffin et al. [9] and Zener[10] discussed the basic theories on GP with engineering application in their books. Another famous book on GP and its application appeared in 1976 (Beightler et al., [11]).The most remarkable property of GP is that a problem with highly nonlinear constraints can be transformed equivalently into a problem with only linear constraints. In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. The idea of impreciseness (fuzziness) in GP i.e. fuzzy geometric programming was proposed by Cao [12]. Ojha et al. [14] used binary number for splitting the cost coefficients, constraints coefficient and exponents and then solved it by GP technique. A solution method of posynomial geometric programming with interval exponents and coefficients was developed by Liu [15]. In 2015, Dey and Roy [16]optimized shape design of structural model with imprecise coefficient by parametric geometric programming .Islam and Roy [17] used FGP to solve a fuzzy EOQ modelwith flexibility and reliability consideration and demand dependent unit production cost a space constraint. FGP method is rarely used to solve the structural optimization problem. Dey and Roy [18] solved two-bar truss non-linear problem using Intuitionistic fuzzy Optimization Technique. But still there are enormous scopes to develop a fuzzy structural optimization model through fuzzy geometric programming (FGP).The parameter used in the GP problem may not be fixed. It is more fruitful to use fuzzy parameter instead of crisp parameter. In that case we can introduce the concept of fuzzy GP technique in parametric form.

In this paper we are making an approach to solve single-objective structural model using parameterized p-norms based fuzzy geometric programming technique. In this structural model formulation, the objective function is to minimize weight of the truss; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. The test problem includes a two-bar planar truss subjected to a single load condition. This approximation approach is used to solve this single-objective structural optimization model.

The remainder of this paper is organized in the following way. In section 2, we discuss about single objective structural optimization model. In section 3, we discuss the mathematical Prerequisites. In section 4, we propose the technique to solve single-objective non-linear programming problem using p-norms based fuzzy optimization. In section 5, apply p-norms based fuzzy optimization technique to solve single-objective structural model and numerical illustration is given. Finally we draw conclusions in section 6.

II. Mathematical Form of a Single – Objective Structural Optimization Model

In sizing optimization problems the aim is to minimize a single objective function, usually the weight of the structure, under certain behavioural constraints on stress and displacements. The design variables are most frequently chosen to be dimensions of the cross-sectional areas of the members of the structure. Due to fabrication limitations the design variables are not continuous but discrete since cross-sections belong to a certain set. A discrete structural optimization problem can be formulated in the following form

$$Minimize \quad f(x) \tag{1}$$

$$Subject\ to\ g_i(x) \le 0, \quad i = 1,2,.......,m$$

$$A_j \in R^d, \quad j = 1,2,.......,n$$

where $f(A)$ represents objective function, $g(A)$ is the behavioural constraint, m and n are the number of constraints and design variables, respectively. A given set of discrete values is expressed by R^d and design variables A_j can take values only from this set.In this paper, objective function is taken as

$$f(A) = \sum_{i=1}^{m} \rho_i A_i l_i \tag{2}$$

and constraints are chosen to be stress of structures

$$g_i(A) = \frac{\sigma_i}{\sigma_i^0} - 1 \le 0 \quad i = 1,2,...,m \tag{3}$$

where ρ_i and l_i are weight of unit volume and length of i^{th} element, respectively, m is the number of the structural elements, σ_i and σ_i^0 are the i^{th} stress and allowable stress, respectively.

III. Prerequisite Mathematics

A. *Fuzzy Set*

Let X is a set (space), with a generic element of X denoted by x , that is $X(x)$ .Then a Fuzzy set (FS) is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$

where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of FS $\tilde{A}$. $\mu_{\tilde{A}}(x)$ is the degree of membership of the element x to the set $\tilde{A}$.

B. *α -Level Set or α -cut of a Fuzzy Set*

The α -level set of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values greater than or equal to α i.e. $\tilde{A} = \{x : \mu_{\tilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

C. *P-norm Generalized Triangular Fuzzy Number)*

A fuzzy number $\tilde{a}_P = \langle (a_1, a_2, a_3; w_a) \rangle_p$ is said to be p-norm generalized triangular fuzzy number $(GTFN)_p$ if its membership function is defined by

$$\mu_{\tilde{a}}(x)=\begin{cases} w_a\left[1-\left(\dfrac{a_2-x}{a_2-a_1}\right)^p\right]^{1/p} & if \quad a_1\le x\le a_2 \\ w_a\left[1-\left(\dfrac{x-a_3}{a_3-a_2}\right)^p\right]^{1/p} & if \quad a_2\le x\le a_3 \\ 0 & otherwise \end{cases}$$

Where w_a represent the maximum degree of membership satisfy in $0\le w_a\le 1$.Also $a_1\le a_2\le a_3$ and p is a positive integer.

It can be easily observed that when $p=1$ $(GTFN)_p$ reduces to GTFN.

*1) **Remart 1:***

A $(GTFN)_p$, $\tilde{a}_p=<(a_1,a_2,a_3;w_a)>_p$ is said to be positive (i.e $\tilde{a}_p>0$) if and only if $a_1\ge 0$, and atleast one of the values of a_1,a_2,a_3 is not equal to zero.

*2) **Remark 2:***

A $(GTFN)_p$, $\tilde{a}_p=<(a_1,a_2,a_3;w_a)>_p$ is said to be positive (i.e $\tilde{a}_p<0$) if and only if $a_1\le 0$ and atleast one of the values of a_1,a_2,a_3 is not equal to zero.

*3) **Remark 3:***

$\tilde{a}_p$ □ 0 if and only if all the values of a_1,a_2,a_3 are equal to zero.

*4) **Remark 4:***

$\tilde{a}_p$ is said to be non- negative if either $\tilde{a}_p$ □ 0 or $\tilde{a}_p>0$.

D. Arithmatic Operations

The arithmetic operations over $(GTFN)_p$ are defined as follows

Let $\tilde{a}_p=<(a_1,a_2,a_3;w_a)>_p$ and $\tilde{b}_p=<(b_1,b_2,b_3;w_b)>_p$ be $(GTFN)_p$,

Then

1) $\tilde{a}_p+\tilde{b}_p=<(a_1+b_1,a_2+b_2,a_3+b_3;\min(w_a,w_b))>_p$.

2) $\tilde{a}_p-\tilde{b}_p=<(a_1-b_1,a_2-b_2,a_3-b_3;\min(w_a,w_b))>_p$.

$$3)\ \lambda\tilde{a}_p=\begin{cases}<(\lambda a_1,\lambda a_2,\lambda a_3;w_a)>_p & if\ \lambda>0\\ <(\lambda a_3,\lambda a_2,\lambda a_1;w_a)>_p & if\ \lambda<0\end{cases}$$

4) $\tilde{a}_p.\tilde{b}_p=$

$$\begin{cases}<(a_1b_1,a_2b_2,a_3b_3;\min(w_a,w_b))>_p & if\,(\tilde{a}_p>0,\tilde{b}_p>0)\\ <(a_1b_3,a_2b_2,a_3b_1;\min(w_a,w_b))>_p & if\,(\tilde{a}_p<0,\tilde{b}_p>0)\\ <(a_3b_3,a_2b_2,a_1b_1;\min(w_a,w_b))>_p & if\,(\tilde{a}_p<0,\tilde{b}_p<0)\end{cases}$$

5) $\tilde{a}_p/\tilde{b}_p=$

$$\begin{cases}<(a_1/b_3,a_2/b_2,a_3/b_1;\min(w_a,w_b))>_p & if\,(\tilde{a}_p>0,\tilde{b}_p>0)\\ <(a_1/b_1,a_2/b_2,a_3/b_3;\min(w_a,w_b))>_p & if\,(\tilde{a}_p<0,\tilde{b}_p>0)\\ <(a_3/b_1,a_2/b_2,a_1/b_3;\min(w_a,w_b))>_p & if\,(\tilde{a}_p<0,\tilde{b}_p<0)\end{cases}$$

IV. MATHEMATICAL ANALYSIS

1) Geometric Programming Method

A geometric program (GP) is a type of mathematical optimization problem characterized by objective and constraint functions that have a special form. GP is a methodology for solving algebraic non-linear optimization problems. Also linear programming is a subset of a geometric programming .The theory of geometric programming was initially developed about three decades ago and culminated in the publication of the seminal text in this area by Duffin, Peterson, and Zener [18].

The general constrained Primal Geometric Programming problem is as follows

$$Minimize\ \ f_0(x)=\sum_{t=1}^{T_0}c_{0t}\prod_{j=1}^{n}x_j^{a_{0tj}} \tag{4}$$

Subject to

$$f_i(x)=\sum_{t=1}^{T_m}c_{it}\prod_{j=1}^{n}x_j^{a_{itj}}\le b_i;\quad i=1,2,3,.......,m$$

$$x_j>0,\qquad j=1,2,..........,n.$$

Here $c_{0t}>0$ and a_{0tj} be any real number. The objective function contains T_0 terms and T_i terms in the inequality constraints. Here the coefficient of each term is positive.So it is a constrained posynomial geometric programming problem. Let $T=T_0+T_1+........+T_i$ be the total number of terms in the primal program. The degree of difficulty (DD) is defined as DD = Total no. of terms – (Total no. of variables -1) $=T-(n+1)$ .The dual problem (with the objective function, $d(w)$ where $w\equiv\{w(w_{it}),\forall i=0,1,2......,m;t=1,2,.....T_i\}$ is the decision vector) of the geometric programming problem (4) for the general posynomial case is as follows

$$Maximize\ \ d(w)=\prod_{t=1}^{T_0}\left(\frac{c_{0t}}{w_{0t}}\right)^{w_{0t}}\prod_{i=1}^{m}\prod_{t=1}^{T_i}\left(\frac{c_{it}\sum w_{it}}{b_iw_{it}}\right)^{w_{it}} \tag{5}$$

Subject to

$$\sum_{t=1}^{T_0}w_{0t}=1\ , \qquad \text{(Normality condition)}$$

$\sum_{i=0}^{m}\sum_{t=1}^{T_i} a_{itj} w_{it} = 0 \quad for\ j = 1,2,......,n.$ (Orthogonality Condition)

$w_{it} > 0 \quad \forall i = 0,1,........,m;\ t = 1,2,........T_i.$

For a primal problem with M variables, $T_0 + T_1 + ... + T_i$ terms and n constraints, the dual problem consists of $T_0 + T_1 + ... + T_i$ variables and $m+1$ constraint. The relation between these problems, the optimality has been shown to satisfy

$$c_{0t}\prod_{j=1}^{n} x_j^{a_{0tj}} = d^*(w^*) \times w_{0t}^* \quad t = 1,2,3,...,T_i \tag{6}$$

$$c_{it}\prod_{j=1}^{n} x_n^{a_{itj}} = \frac{w_{it}^*}{\sum_{t=1}^{T_i} w_{it}^*} \quad i = 1,2,3,....,m;\ t = 1,2,3,...,T_i \tag{7}$$

Taking logarithms in (6) and (7) and putting $t_j = \log x_j$ for $j = 1,2,..........,n.$ we shall get a system of linear equations of t_j ($j = 1,2,..........,n.$).We can easily find primal variables from the system of linear equations.

Case I: For $T \geq n+1$,the dual program presents a system of linear equations for the dual variables where the number of linear equations is either less than or equal to the number of dual variables. A solution vector exists for the dual variable (Beightler and Philips [20]).

Case II: For $T < n+1$,the dual program presents a system of linear equations for the dual variables where the number of linear equation is greater than the number of dual variables. In this case, generally, no solution vector exists for the dual variables. However, one can get an approximate solution vector for this system using either the least squares or the linear programming method.

2) *Fuzzy Geometric Programming Problem*

The formulation of fuzzy geometric programming with fuzzy parameters can be stated as follows

$$Find \quad x = (x_1, x_2, x_3..........x_n)^T \tag{8}$$

so as to

$$Minimize \quad f_0(x) = \sum_{t=1}^{T_0} \tilde{c}_{0t} \prod_{j=1}^{n} x_j^{a_{0tj}}$$

Such that

$$f_i(x) = \sum_{t=1}^{T_i} \tilde{c}_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \precsim \tilde{b}_i \quad for \quad i = 1,2,.........,m$$

$$x_j > 0 \qquad for \quad j = 1,2,........,n$$

where $\tilde{c}_{0t}, \tilde{c}_{it}, \tilde{b}_i$ are p-norm based generalised positive triangular fuzzy number. a_{0tj} and a_{itj} are real numbers for all i,t,j .

Let

$\tilde{c}_{0t} \cong <(c_{10t}, c_{20t}, c_{30t}; w)>_p \quad (1 \leq t \leq T_0)$

$\tilde{c}_{it} \cong <(c_{1it}, c_{2it}, c_{3it}; w)>_p \quad (1 \leq t \leq T_i, 1 \leq i \leq m)$

$\tilde{b}_i \cong <(b_{1i}, b_{2i}, b_{3i}; w)>_p \quad (1 \leq i \leq m)$

be p-norm based triangular fuzzy numbers with membership functions

$$\mu_{\tilde{c}_{0t}}(x) = \begin{cases} w\left[1-\left(\dfrac{c_{20t}-x}{c_{20t}-c_{10t}}\right)^p\right]^{1/p} & if \ c_{10t} \leq x \leq c_{20t} \\ w & if \quad x = a_2 \\ w\left[1-\left(\dfrac{x-c_{30t}}{c_{30t}-c_{20t}}\right)^p\right]^{1/p} & if \ c_{20t} \leq x \leq c_{30t} \\ 0 & otherwise \end{cases}$$

$$\mu_{\tilde{c}_{it}}(x) = \begin{cases} w\left[1-\left(\dfrac{c_{2it}-x}{c_{2it}-c_{1it}}\right)^p\right]^{1/p} & if \ c_{1it} \leq x \leq c_{2it} \\ w & if \quad x = a_2 \\ w\left[1-\left(\dfrac{x-c_{3it}}{c_{3it}-c_{2it}}\right)^p\right]^{1/p} & if \ c_{2it} \leq x \leq c_{3it} \\ 0 & otherwise \end{cases}$$

$$\mu_{\tilde{b}_i}(x) = \begin{cases} w\left[1-\left(\dfrac{b_{2i}-x}{b_{2i}-b_{1i}}\right)^p\right]^{1/p} & if \ c_{1it} \leq x \leq c_{2it} \\ w & if \quad x = b_2 \\ w\left[1-\left(\dfrac{x-c_{3i}}{c_{3i}-c_{2i}}\right)^p\right]^{1/p} & if \ b_{2i} \leq x \leq c_{3i} \\ 0 & otherwise \end{cases}$$

where the functions $f_{c_{0tl}} : [c_{10t}(\alpha), c_{20t}] \to [0,w]$, $f_{citl} : [c_{1it}, c_{2it}] \to [0,w]$, $f_{bitl} : [b_{1i}, b_{2i}] \to [0,w]$ Where $w \in [0,1]$ are continuous and non-decreasing and $f_{c_{0tr}} : [c_{20t}(\alpha), c_{30t}] \to [0,w]$, $f_{c_{itr}} : [c_{20t}(\alpha), c_{30t}] \to [0,w]$, $f_{c_{itr}} : [c_{2it}, c_{3it}] \to [0,w]$,

Where $w \in [0,1]$ are continuous and non-increasing function and w is called maximum membership degree.

Here $\alpha -$ cut of $\tilde{c}_{0t}, \tilde{c}_{it}, \tilde{b}_i$ are given by

$$c_{0t}(\alpha) = [c_{0tL}(\alpha), c_{0tR}(\alpha)]$$

$$= \left[c_{20t} - \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(c_{20t}-c_{10t}),\ c_{30t} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(c_{30t}-c_{20t}) \right]$$

$$c_{it}(\alpha) = [c_{itL}(\alpha), c_{itR}(\alpha)]$$

$$= \left[c_{2it} - \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(c_{2it}-c_{1it}),\ c_{3it} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(c_{3it}-c_{2it}) \right]$$

$$Find \quad x = (x_1, x_2, x_3, ..., x_n)^T \tag{9}$$

So as to

$$Minimize \quad f_0^R = \sum_{t=1}^{T_0}\left[c_{30t} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(c_{30t}-c_{20t}) \right]\prod_{j=1}^{n} x_j^{a_{0tj}}$$

such that

$$f_i(x) \equiv \sum_{t=1}^{T_i} \left[\frac{c_{30t} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (c_{30t}-c_{20t})}{b_{2i} - \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (b_{2i}-b_{1i})} \right] \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$$

$for \quad i = 1,2,................,m$

$x_j > 0 \quad for \quad j = 1,2,3...................n, \quad \alpha \in [0,1]$

Using $\alpha-$cut of the p-norm generalised triangular fuzzy number coefficients the above problem reduces to

$Find \;\; x = (x_1, x_2, x_3, ...x_n)^T$ (10)

so as to

$$Minimize = f_0(x) = \sum_{t=1}^{T_0} [c_{0tL}(\alpha), c_{0tR}(\alpha)] \prod_{J=1}^{n} x_j^{a_{0tj}}$$

Such that

$$f_i(x) \equiv \sum_{t=1}^{T_i} [c_{itL}(\alpha), c_{itR}(\alpha)] \prod_{j=1}^{n} x_j^{a_{itj}} \le [b_{iL}(\alpha), b_{iR}(\alpha)]$$

$for\, i = 1,2,.......m$

a_{0tj} and a_{itj} re real numbers for all i,t,j.

The above problem is equivalent to the sub-problem

$Find \quad x = (x_1, x_2, x_3, ..., x_n)^T$ (11)

so as to

$$Minimize \;\; f_0^L(x) = \sum_{t=1}^{T_0} \left[c_{20t} - \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (c_{20t}-c_{10t}) \right] \prod_{j=1}^{n} x_j^{a_{0tj}}$$

$such \;\; that$

$$f_i(x) \equiv \sum_{t=1}^{T_i} \left[\frac{c_{2it} - \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (c_{20t}-c_{10t})}{b_{3i} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (b_{3i}-b_{2i})} \right] \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$$

$for \;\; i = 1,2,..........,m$

$x_j > 0 \quad for \quad j = 1,2,....,n, \; \alpha \in [0,w]$

a_{0tj} and a_{itj} are real numbers for all i,t,j.

$Find \quad x = (x_1, x_2, x_3, ..., x_n)^T$ so as to (12)

$$Minimize \quad f_0^R = \sum_{t=1}^{T_0} \left[c_{30t} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (c_{30t}-c_{20t}) \right] \prod_{j=1}^{n} x_j^{a_{0tj}}$$

$such \; that$

$$f_i(x) \equiv \sum_{t=1}^{T_i} \left[\frac{c_{30t} + \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (c_{30t}-c_{20t})}{b_{2i} - \left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p} (b_{2i}-b_{1i})} \right] \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$$

$for \quad i = 1,2,.....,m$

$x_j > 0 \quad for \quad j = 1,2,3.....n, \quad \alpha \in [0,1]$

a_{0tj} and a_{itj} are real numbers for all i,t,j.

Now solving above two sub problem by geometric programming technique we can get the upper and lower bound of objective function for each $\alpha \in [0,1]$.

IV. NUMERICAL ILLUSTRATION

A well-known two-bar [17] planar truss structure is considered. The design objective is to minimize weight of the structural $WT(A_1, A_2, y_B)$ of a statistically loaded two-bar planar truss subjected to stress $\sigma_i(A_1, A_2, y_B)$ constraints on each of the truss members $i = 1,2$.

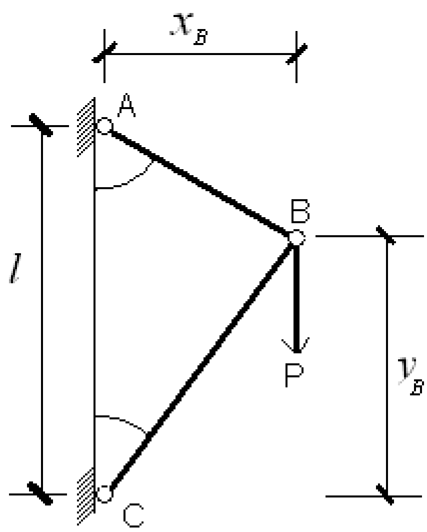

Fig. 1 Design of the two-bar planar truss

The single-objective structural model can be expressed as

$$Minimize\; WT(A_1, A_2, y_B) = \rho\left(A_1\sqrt{x_B^2 + (l-y_B)^2} + A_2\sqrt{x_B^2 + y_B^2}\right) \quad (13)$$

$such \;\; that$

$$\sigma_{AB}(A_1, A_2, y_B) \equiv \frac{P\sqrt{x_B^2 + (l-y_B)^2}}{lA_1} \le [\sigma_{AB}^T]$$

$$\sigma_{BC}(A_1, A_2, y_B) \equiv \frac{P\sqrt{x_B^2 + y_B^2}}{lA_2} \le [\sigma_{BC}^C]$$

$0.5 \le y_B \le 1.5$; $A_1 > 0, A_2 > 0$;

The input data for structural optimization problem (13) is given as follows

$Nodal \;\; load \;\; (\tilde{P}) \cong \; < (90,100,110; 0.8) >_p \; KN,$

$Volume \; Density \; (\tilde{\rho}) \cong \; < (7.6, 7.7, 7.8; 0.8) >_p \;\; KN/m^3,$

$Length \;\; (l) = 2m;$

$Width \;\; (\tilde{x}_B) \cong \; < (.9, 1, 1.1, 0.8) >_p \; Mpa;$

$Allowable \; compressive \; stress \; \tilde{\sigma}_c \cong \; < (90,100,110; 0.8) >_p \;\; Mpa;$

Allowable tensile stress $\tilde{\sigma}_T \cong <(145,150,155;0.8)>_p$ *Mpa;*

y *coordinate of node* $B(y_B)=(0.5\le y_B\le 1.5)$

Solution:

The non-linear structural optimization problem of two bar truss is

$$\text{Minimize } WT(A_1,A_2,y_B)=$$

$$<(7.6,7.7,7.8;0.8)>_p\left(A_1\sqrt{\left((<(.9,1,1.1,0.8)>_p)^2+(2-y_B)^2\right)}+A_2\sqrt{\left((<(.9,1,1.1,0.8)>)^2+y_B^2\right)}\right) \quad (14)$$

Such that

$$\sigma_{AB}(A_1,A_2,y_B)\equiv\frac{<(90,100,110;0.8)>_p\sqrt{\left((<(.9,1,1.1,0.8)>_p)^2+(2-y_B)^2\right)}}{2A_1}$$

$$\le\left(<(145,150,155;0.8)>_p\right),$$

$$\sigma_{BC}(A_1,A_2,y_B)\equiv\frac{<(90,100,110;0.8)>_p\sqrt{\left((<(.9,1,1.1,0.8)>_p)^2+y_B^2\right)}}{2A_2}$$

$$\le\left(<(90,100,110;0.8)>_p\right),$$

$$_{AB}(A_1,A_2,y_B)\equiv\frac{<(90,100,110;0.8)>_p\sqrt{\left((<(.9,1,1.1,0.8)>_p)^2+(2-y_B)^2\right)}}{2A_1}$$

$$\sigma\le\left(<(145,150,155;0.8)>_p\right),$$

$$0.5\le y_B\le 1.5,\quad A_1>0,\ A_2>0$$

The $\alpha-$ cut of $\tilde{\rho},\tilde{x}_B,\tilde{P},\tilde{\sigma}_t,\tilde{\sigma}_c$ are given by

$$\rho=\left[7.7-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1),7.8+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right] \quad (15)$$

$$x_B=\left[7.7-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1),1.1+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right]$$

$$P=\left[100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10),110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)\right]$$

$$\sigma_t=\left[150-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5),155+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5)\right]$$

$$\sigma_c=\left[100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1),110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)\right]$$

$$\rho=\left[7.7-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1),7.8+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right]$$

$$x_B=\left[7.7-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1),1.1+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right]$$

$$P=\left[100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10),110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)\right]$$

$$\sigma_t=\left[150-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5),155+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5)\right]$$

$$\sigma_c=\left[100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1),110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)\right]$$

Using $\alpha-$ cut above problem (15) is reduced to the two sub problems

$$\text{Minimize } WT^L(A_1,A_2,y_B)=\left[7.7-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right] \quad (16)$$

$$\left(A_1\sqrt{\left(\left(1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)^2+(2-y_B)^2\right)}+A_2\sqrt{\left(\left(1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)^2+y_B^2\right)}\right)$$

subject to $\sigma_{AB}(A_1,A_2,y_B)\equiv$

$$\frac{\left[\dfrac{100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{155+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5)}\right]\sqrt{\left(\left(1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)+(2-y_B)^2\right)}}{2A_1}\le 1,$$

$\sigma_{BC}(A_1,A_2,y_B)\equiv$

$$\frac{\left[\dfrac{100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}\right]\sqrt{\left(\left(1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)+y_B^2\right)}}{2A_2}\le 1,$$

$A_1,A_2>0;\quad 0.5\le y_B\le 1.5 \qquad \alpha\in[0,w]$ and

$$\text{Minimize } WT^R(A_1,A_2,y_B)=\left[7.8+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right]$$

$$\left(A_1\sqrt{\left(\left(1.1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)^2+(2-y_B)^2\right)}+A_2\sqrt{\left(\left(1.1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)^2+y_B^2\right)}\right) \quad (17)$$

such that $\sigma_{AB}(A_1,A_2,y_B)\equiv$

$$\frac{\left[\dfrac{110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{150-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5)}\right]\sqrt{\left(\left(1.1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)+(2-y_B)^2\right)}}{2A_1}\le 1,$$

$$\sigma_{BC}(A_1, A_2, y_B) \equiv \frac{\left[\dfrac{110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}\right]\sqrt{\left(\left(1.1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right)+y_B{}^2\right)}}{2A_2} \le 1,$$

$A_1, A_2 > 0;\quad 0.5 \le y_B \le 1.5 \qquad \alpha \in [0, w]$

To apply Geometric Programming Technique we may consider any of the sub problems as

$$\textit{Minimize}\ WT\ (A_1, A_2, y_B) =$$

$$M\left(A_1\sqrt{\left(L(\alpha)\right)^2+(2-y^2{}_B)}+A_2\sqrt{\left(L(\alpha)\right)^2+(2-y^2{}_B)}\right) \qquad (18)$$

subject to

$$\sigma_{AB}(A_1, A_2, y_B) \equiv \frac{N\sqrt{\left(L(\alpha)\right)^2+(2-y^2{}_B)}}{2A_1} \le 1,$$

$$\sigma_{BC}(A_1, A_2, y_B) \equiv \frac{S\sqrt{\left(L(\alpha)\right)^2+y^2{}_B}}{2A_2} \le 1,$$

$A_1 > 0, A_2 > 0;\quad 0.5 \le y_B \le 1.5$

Let $(L(\alpha))^2 + (2-y_B)^2 \le A_3^2$ *and* $(L(\alpha)) + y_B^2 \le A_4^2$

Then above problem (18) can be written as

$$\textit{Minimize}\ WT(A_1, A_2, y_B) = M(A_1A_3 + A_2A_4) \qquad (19)$$

subject to

$$\sigma_{AB}(A_1, A_2, y_B) \equiv \frac{NA_3}{2A_1} \le 1,$$

$$\sigma_{BC}(A_1, A_2, y_B) \equiv \frac{SA_4}{2A_2} \le 1,$$

$$((L(\alpha))^2+4)A_3^{-2} - 4y_BA_3^{-2} + y_B^2A_3^{-2} \le 1,$$

$$2\left(\left(L(\alpha)\right)^2+4\right)A_3^{-2} - 4y_BA_3^{-2} + y_B^2A_3^{-2} \le 1,$$

$$2\left(L(\alpha)\right)^2A_4^{-2} + y_B^2A_4^{-2} \le 1,$$

$0.5 \le y_B \le 1.5 \quad A_1 > 0, A_2 > 0, A_3 > 0, A_4 > 0$

This is a signomial Geometric Programming Problem with DD=9-(5+1)=3

The dual formulation is

$$\textit{Maximize}\ d_{WT}(w_{01}, w_{02}, w_{11}, w_{21}, w_{31}, w_{32}, w_{33}, w_{41}, w_{41}) =$$

$$\left(\frac{M}{w_{01}}\right)^{w_{01}}\left(\frac{M}{w_{02}}\right)^{w_{02}}\left(\frac{N}{2w_{11}}\right)^{w_{11}}\left(\frac{S}{2w_{11}}\right)^{w_{21}}$$

$$\left(\frac{\left(\left(L(\alpha)\right)^2+4\right)^2\left(w_{31}-w_{32}+w_{33}\right)}{w_{31}}\right)^{w_{31}}\left(\frac{4(w_{31}-w_{32}+w_{33})}{w_{32}}\right)^{-w_{32}}$$

$$\left(\frac{(w_{31}-w_{32}+w_{33})}{w_{33}}\right)^{w_{33}}\left(\frac{\left(L(\alpha)\right)^2\left(w_{41}+w_{42}\right)}{w_{41}}\right)^{w_{41}}\left(\frac{\left(w_{41}+w_{42}\right)}{w_{42}}\right)^{w_{42}} \qquad (20)$$

such that

$$w_{01} + w_{02} = 1,\ \ w_{01} - w_{11} = 0,\ \ w_{02} - w_{21} = 0,$$

$$w_{01} + w_{11} - 2w_{31} + 2w_{32} - 2w_{33} = 0,$$

$$w_{02} + w_{21} - 2w_{41} - 2w_{42} = 0,\ -w_{32} + 2w_{33} + 2w_{42} = 0,$$

The constraints of (20) forms a system of six linear equations with nine unknowns .So the system has infinite number of solutions .However the problem is to select the optimal dual variables $w_{01}, w_{02}, w_{11}, w_{21}, w_{31}, w_{32}, w_{33}, w_{41}, w_{42}$.

We have

$$w_{01} = w_{11} = w_{31} - w_{32} + w_{33}, w_{02} = w_{21} = 1 - w_{31} + w_{32} - w_{33},$$

$$w_{41} = 1 - w_{31} + 0.5w_{32}, w_{42} = 0.5w_{32} - w_{33}$$

Substituting $w_{01}, w_{02}, w_{11}, w_{21}, w_{41}, w_{42}$ in the dual formulation we get $\textit{Maximize}\ \ d_{WT}(w_{31}, w_{32}, w_{33}) =$

$$\left(\frac{M}{w_{31}-w_{32}+w_{33}}\right)^{(w_{31}-w_{32}+w_{33})}\left(\frac{M}{1-w_{31}+w_{32}-w_{33}}\right)^{(1-w_{31}+w_{32}-w_{33})}\left(\frac{N}{2(w_{31}-w_{32}+w_{33})}\right)^{(w_{31}-w_{32}+w_{33})}$$

$$\left(\frac{S}{2(1-w_{31}+w_{32}-w_{33})}\right)^{(1-w_{31}+w_{32}-w_{33})}\left(\frac{((L(\alpha))^2+4)\left(w_{31}-w_{32}+w_{33}\right)}{w_{31}}\right)^{w_{31}}$$

$$\left(\frac{4(w_{31}-w_{32}+w_{33})}{w_{32}}\right)^{-w_{32}}\left(\frac{(w_{31}-w_{32}+w_{33})}{w_{33}}\right)^{w_{33}}$$

$$\left(\frac{\left(L(\alpha)\right)^2\left(w_{41}+w_{42}\right)}{1-w_{31}+0.5w_{32}}\right)^{(1-w_{31}+0.5w_{32})}\left(\frac{w_{41}+w_{42}}{0.5w_{32}-w_{33}}\right)^{(0.5w_{32}-w_{33})} \qquad (21)$$

To find the optimal w_{31}, w_{32}, w_{33} which maximizes the dual $d_{WT}(w_{31}, w_{32}, w_{33})$ we take logarithm of both sides of (21) and get

$$\log d_{WT}(w_{31}, w_{32}, w_{33}) =$$

$$(w_{31}-w_{32}+w_{33})\log M - (w_{31}-w_{32}+w_{33})\log(w_{31}-w_{32}+w_{33})$$

$$+(1-w_{31}+w_{32}-w_{33})\log M - (1-w_{31}+w_{32}-w_{33})\log(1-w_{31}+w_{32}-w_{33})$$

$$+(w_{31}-w_{32}+w_{33})\log N - (w_{31}-w_{32}+w_{33})\log(2(w_{31}-w_{32}+w_{33}))$$

$$+(1-w_{31}+w_{32}-w_{33})\log S - (1-w_{31}+w_{32}-w_{33})\log(2(1-w_{31}+w_{32}-w_{33}))$$

$$+w_{31}\log\left((Q^2+4)\left(w_{31}-w_{32}+w_{33}\right)\right) - w_{31}\log w_{31}$$

$$-w_{32}\log\left(4\left(w_{31}-w_{32}+w_{33}\right)\right) + w_{32}\log w_{32} + w_{33}\log(w_{31}-w_{32}+w_{33})$$

$$-w_{33}\log w_{33} + (1-w_{31}+0.5w_{32})\log\left(Q^2(w_{41}+w_{42})\right)$$

$$-(1-w_{31}-0.5w_{32})\log(1-w_{31}+0.5w_{32}) + (0.5w_{32}-w_{33})\log(w_{41}+w_{42})$$

$$-(0.5w_{32}-w_{33})\log(0.5w_{32}-w_{33})$$

Differentiating partially with respect to w_{31}, w_{32} and w_{33} respectively and equating to zero we get

$$\left(1-w_{31}+0.5w_{32}\right)N\left(\left(L(\alpha)\right)^2+4\right) - Sw_{31}\left(L(\alpha)\right)^2 = 0,$$

$$4N\left(1-w_{31}+0.5w_{32}\right)^{0.5}(0.5w_{32}-w_{33})^{0.5} - \left(L\left(\alpha\right)\right)Sw_{32} = 0,$$

and

$$N(0.5w_{32}-w_{33}) - Sw_{33} = 0$$

$$\frac{\partial^2 \log d_{WT}(w_{31}, w_{32}, w_{33})}{\partial^2 w_{31}} = -\left[\frac{1}{w_{31}} + \frac{1}{1-w_{31}+0.5w_{32}}\right]$$

$$\frac{\partial^2 \log d_{WT}(w_{31}, w_{32}, w_{33})}{\partial^2 w_{33}w_{31}} = 0$$

$$\frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{32}} =$$

$$-\frac{3}{w_{31}-w_{32}+w_{33}}+\frac{0.5}{1-w_{31}+0.5w_{32}}+\frac{1}{1-w_{31}+w_{32}-w_{33}}$$

$$\frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}w_{32}} =$$

$$-\frac{3}{w_{31}-w_{32}+w_{33}}+\frac{0.5}{0.5w_{32}-w_{33}}$$

$$\frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{33}} = 0$$

$$\frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}} = -\frac{1}{w_{33}}-\frac{1}{0.5w_{32}-w_{33}}$$

It is to be noted that for optimum dual variable $w_{31}^*, w_{32}^*, w_{33}^*$ the Hassian matrix

$$\begin{vmatrix} \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{32}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{33}} \\ \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}w_{33}} \\ \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}w_{32}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}} \end{vmatrix}$$

must be negative definite .

i.e. $\left|\frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}}\right| < 0,$

$$\begin{vmatrix} \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{32}} \\ \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}} \end{vmatrix} > 0$$

$$\begin{vmatrix} \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{32}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{31}w_{33}} \\ \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{32}w_{33}} \\ \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}w_{31}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}w_{32}} & \frac{\partial^2 \log d_{WT}(w_{31},w_{32},w_{33})}{\partial^2 w_{33}} \end{vmatrix} < 0$$

Now from the primal dual relation

$$MA_1A_3 = w_{01}^* d^*\left(w_{01}^*, w_{02}^*, w_{11}^*, w_{21}^*, w_{31}^*, w_{32}^*, w_{33}^*, w_{41}^*, w_{42}^*\right)$$

$$MA_2A_4 = w_{02}^* d^*\left(w_{01}^*, w_{02}^*, w_{11}^*, w_{21}^*, w_{31}^*, w_{32}^*, w_{33}^*, w_{41}^*, w_{42}^*\right)$$

$$\frac{NA_3}{2A_1} = \frac{w_{11}^*}{w_{11}^*} = 1;\ \frac{SA_4}{2A_2} = \frac{w_{21}^*}{w_{21}^*} = 1$$

$$\left(L^2+4\right)A_3^{-2} = \frac{w_{31}^*}{w_{31}^*-w_{32}^*+w_{33}^*}$$

$$4y_B x_3^{-2} = \frac{w_{32}^*}{w_{31}^*-w_{32}^*+w_{33}^*}$$

$$y_B^2 A_3^{-2} = \frac{w_{33}^*}{w_{31}^*-w_{32}^*+w_{33}^*}$$

$$L^2 A_4^{-2} = \frac{w_{41}^*}{w_{41}^*+w_{42}^*} = \frac{1-w_{31}+0.5w_{32}}{1+w_{32}-w_{31}-w_{33}}$$

$$y_B^2 A_4^{-2} = \frac{w_{42}^*}{w_{41}^*+w_{42}^*} = \frac{0.5w_{32}-w_{33}}{1+w_{32}-w_{31}-w_{33}}$$

we will get optimal solution for A_1^*, A_2^*.

Now for $\alpha \in [0,1]$

$$M(\alpha)=\left[7.7-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right], L(\alpha)=\left[1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right]$$

$$N(\alpha)=\left[\frac{100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{155+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5)}\right], S(\alpha)=\left[\frac{100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}\right]$$

and for $\alpha \in [0,1]$

$$M(\alpha)=\left[7.8-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right], L(\alpha)=\left[1-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(0.1)\right]$$

$$N(\alpha)=\left[\frac{110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{150-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(5)}\right], S(\alpha)=\left[\frac{110+\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}{100-\left[1-\left(\frac{\alpha}{w}\right)^p\right]^{1/p}(10)}\right]$$

the above problem gives the left and right spread of weight interval.

VI.TABLE I

OPTIMIZED RESULT OF DESIGN VARIABLES OF TRUSS FOR $w = 0.8$.

Level of Possibility or Degree of Uncertainty	Left Spread of Design Variables	Norm P=1	Norm P=2	Right Spread of Design Variables	Norm P=1	Norm P=2
$\alpha = 0.0$	WT^L	9.9173	9.917	WT^R	19.272	19.271
	A_1^L	0.4546	0.4546	A_1^R	0.6975	0.6974
	A_2^L	0.5178	0.5178	A_2^R	.8708	0.8708
	y_B^L	0.8571	0.8571	y_B^R	0.8163	0.8163
$\alpha = 0.1$	WT^L	10.258	9.938	WT^R	18.645	19.233
	A_1^L	0.4653	0.4552	A_1^R	0.6825	0.6965
	A_2^L	0.5327	0.5187	A_2^R	0.8463	0.8692
	y_B^L	0.8539	0.8569	y_B^R	0.8202	0.8165
$\alpha = 0.2$	WT^L	10.610	10.002	WT^R	18.038	19.110
	A_1^L	0.4762	0.4573	A_1^R	0.6678	0.6936
	A_2^L	0.5481	0.5216	A_2^R	0.8226	0.8645
	y_B^L	0.8506	0.8563	y_B^R	0.8241	0.8173
$\alpha = 0.3$	WT^L	10.973	10.115	WT^R	17.451	18.903
	A_1^L	0.4874	0.4608	A_1^R	0.6532	0.6886
	A_2^L	0.5639	0.5265	A_2^R	0.7995	0.8564
	y_B^L	0.8473	0.8552	y_B^R	0.8279	0.8186
$\alpha = 0.4$	WT^L	11.348	10.283	WT^R	16.882	18.600
	A_1^L	0.4987	0.4661	A_1^R	0.6390	0.6814
	A_2^L	0.5801	0.5338	A_2^R	0.7777	0.8446
	y_B^L	0.8440	0.8537	y_B^R	0.8316	0.8205
$\alpha = 0.5$	WT^L	11.735	10.522	WT^R	16.331	18.185
	A_1^L	0.5102	0.4735	A_1^R	0.6250	0.6713
	A_2^L	0.5968	0.5443	A_2^R	0.7554	0.8283
	y_B^L	0.8406	0.8514	y_B^R	0.8353	0.8231
$\alpha = 0.6$	WT^L	12.113	11.370	WT^R	15.797	17.620
	A_1^L	0.5220	0.4841	A_1^R	0.6113	0.6574
	A_2^L	0.6140	0.5592	A_2^R	0.7343	0.8062
	y_B^L	0.8372	0.8483	y_B^R	0.8390	0.8268
$\alpha = 0.7$	WT^L	13.548	11.397	WT^R	11.397	16.811
	A_1^L	0.5339	0.5002	A_1^R	0.5002	0.6372
	A_2^L	0.6316	0.5822	A_2^R	0.5822	0.7743
	y_B^L	0.8337	0.8436	y_B^R	0.8436	0.8321
$\alpha = 0.8$	WT^L	12.974	12.974	WT^R	12.974	14.779
	A_1^L	0.5460	0.5460	A_1^R	0.5460	0.5845
	A_2^L	0.6498	0.6498	A_2^R	0.6498	0.6938
	y_B^L	0.8301	0.8301	y_B^R	0.8301	0.8461

In general the value of α shows that the level of possibility and degree of uncertainty of the obtained information. When the value of α increases, the level of possibility becomes greater and the degree of uncertainty become less. From the above result it is clear that when $\alpha = 0$ the widest interval indicates that objective value definitely lie into this range. On the other hand the possibility level $\alpha = 0.8$ indicates the most possible value of the objective function. In this example the objective value is impossible to fall below 9.917 or exceed 19.272 for p=1,2 and the most possible value lie within 12.974 and 14.779 for p=1,2.

VII. CONCLUSIONS

In this work, a Geometric Programming for Structural Optimization of two bar truss design problem has been discussed. The considered problem is a highly nonlinear and non-exact in nature. Here the parameter is taken as p norm based generalised triangular fuzzy number and Zadeh's extension principle has been used to transform the fuzzy geometric programming problem to a pair of two mathematical programmes. P-norm based fuzzy number can be used in several optimization design problems.

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