An Efficient Algorithm to obtain the optimal solution for Fuzzy Transportation problems

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Abstract:

In this paper we solve the Fuzzy Transportation problems by using a new algorithm namely EAVAM . We introduce an approach for solving a wide range of such problems by using a method which applies it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal or any LR fuzzy number. Thus, some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, into crisp quantities by using our method and then by using the EAVAM, we solve and obtain the solution of the problem. The new method is systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function.

Finally we explain this method with a numerical example.

Keywords: Optimization, Transportation problem, ranking of fuzzy numbers, .Fuzzy sets, Fuzzy transportation problem,

1. Introduction

Transportation models have wide applications in logistics and supply chain for reducing the cost. When the cost coefficients and the supply and demand quantities are known exactly, many algorithms have been developed for solving the transportation problem. But, in the real world, there are many cases that the cost coefficients and the supply and demand quantities are fuzzy quantities.

The transportation problem is one of the oldest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock [3]. Efficient methods of solution derived from the simplex algorithm were developed in 1947, primarily by Dantzig [4] and then by Charnes and Cooper [1]. The transportation problem can be modeled as a standard linear programming problem that can be solved by the simplex method. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row Minima, Column Minima, Matrix Minima or the Vogel's Approximation Method. To get an optimal solution for the transportation problem, we use the MODI method (Modified Distribution Method).Charnes and Cooper [1] developed the Stepping Stone Method, which provides an alternative way of determining the optimal solution The LINDO (Linear Interactive and Discrete Optimization) package handles the transportation problem in explicit equation form and thus solves the problem as standard linear programming problem. Consider m origins (or sources) O_i (i =1,....,m)and n destinations D_j (j = 1,...,n). At each origin O_i, let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j, in order to satisfy the demand for b_j units of the product there. A penalty c_{ij} is associated with transport in a unit of the product from source i to destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under-used capacity, etc. A variable x_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. The transportation problem can be represented as a single objective transportation problem or as a multi-objective transportation problem. Fuzzy transportation problem (FTP)[6] is the problem of minimizing fuzzy valued objective functions with fuzzy source and fuzzy destination parameters. The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problem. Shan Huo Chen [15] introduced the concept of function principle, which is used to calculate the fuzzy transportation cost. The Graded Mean Integration Representation Method, used to defuzzify the fuzzy transportation cost, was also introduced by Shan Huo Chen [14].

In this problem we determine optimal shipping patterns between origins or sources and destinations. Many problems which have nothing to do with transportation have this structure. In many fuzzy decision problems, the data are represented in terms of fuzzy numbers. In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal. Thus, some fuzzy numbers are not directly comparable. Comparing between two or multi fuzzy numbers and ranking such numbers is one of the important subjects, and how to set the rank of fuzzy numbers has been one of the main problems. Several methods are introduced for ranking of fuzzy numbers. Here, we want to use a method which is introduced for ranking of fuzzy numbers, by Basirzadeh et al. [3]. Now, we want to apply this method for the fuzzy transportation problems, where all parameters can be, triangular fuzzy numbers. This method is very easy to understand and to apply. At the end, the optimal solution of a problem can be obtained in a fuzzy number or a crisp number.

2. Terminology

Consider m origins (or sources) O_i (i = 1,...,m) and n destinations D_j (j = 1,...,n). At each origin O_i, let a_i be the amount of a homogeneous product that we want to transport to n destinations D_j, in order to satisfy the demand for b_j units of the product there. A penalty c_{ij} is associated with transportation of a unit of the product from source i to destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered under-used capacity, etc. A variable xij represents the unknown quantity to be transported from origin O_i to destination D_j. However, in the real world, all transportation problems are not single objective linear programming problems.

The mathematical form of the above said problem is as follows

Minimize Z = $\sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij} \quad x_{ij}$ (1)

$\sum_{j=1}^{m} x_{ij} = a_i$, i=1,2,…m.

$\sum_{i=1}^{m} x_{ij} = b_j$, j=1,2,…m.

$x_{ij} \geq 0$, i= 1,2,..m, j=1,2,..m.

where c_{ij} is the cost of transportation of an unit from the i[th] source to the j[th] destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the i[th] origin to j[th] destination. An obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \qquad (2)$$

i.e. assume that total available is equal to the total required. If not true, a fictitious source or destination can be added. It is has a feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij} , in such a way that the total transportation cost is minimum. Mathematically a fuzzy transportation problem can be stated as follows:

Minimize Z = $\sum_{i=1}^{m}\sum_{j=1}^{m} \widetilde{c_{ij}} \quad x_{ij}$ (3)

$\sum_{j=1}^{m} x_{ij} = \tilde{a}_i$, i=1,2,…m.

$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j$, j=1,2,…m. (5)

$x_{ij} \geq 0$, i= 1,2,..m, j=1,2,..m.

in which the transportation costs $\widetilde{c_{ij}}$, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem given in (2-3) to have a solution is that

$$\sum_{i=1}^{n} \tilde{a}_i \approx \sum_{j=1}^{m} \tilde{b}_j \qquad (4)$$

A considerable number of methods presented for fuzzy transportation problem. Some of them are based on ranking of the fuzzy numbers. Some of the methods for ranking of the fuzzy numbers for example, have limitations, are difficult in calculation, or they are non-intuitive, which makes them inefficient in practical applications, especially in the process of decision making. However, in some of these methods, as ones in which fuzzy numbers are compared according to their centeroid point (see [2], [15], [16]), the decision maker does not play any role in the comparison between fuzzy numbers. Nevertheless, there are certain methods in which fuzzy numbers are compared in a parametric manner

Two factors play significant roles in fuzzy decision systems:

1. Contribution of the decision-maker in the decision making process,
2. Simplicity of calculation.

This paper attempts to propose a method for ranking and comparing (2) fuzzy numbers to account for the above-mentioned factors as much as possible.

2.1 Definition

A fuzzy number $\breve{a}$ is a Triangular-Fuzzy number denoted $(a_1,a_2,a_3,)$ where a_1,a_2 and a_3 and it's membership function $\mu_{\tilde{a}}\ (a)$ is given below

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \frac{x-a_3}{a_2-a_3} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Consider the following fuzzy transportation problem (FTP),

(FTP) Minimize: $Z = \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{c}_{ij} \ \ \tilde{x}_{ij}$

Subject to

$\sum_{j=1}^{m} \tilde{x}_{ij} \le \tilde{a}_i$, i=1,2,…m.

$\sum_{i=1}^{m} \tilde{x}_{ij} \ge \tilde{b}_j$, j=1,2,…m.

$\tilde{x}_{ij} \ \ \ge 0$, i= 1,2,..m, j=1,2,..m.

Where $\tilde{a}_i$ = (a_1,a_2,a_3), $\tilde{b}_j$ =(b_1,b_2, b_3) and $\tilde{c}_{ij}$ =(c_{ij},c_{ij},c_{ij}) representing the uncertain supply and demand for the transportation problems.

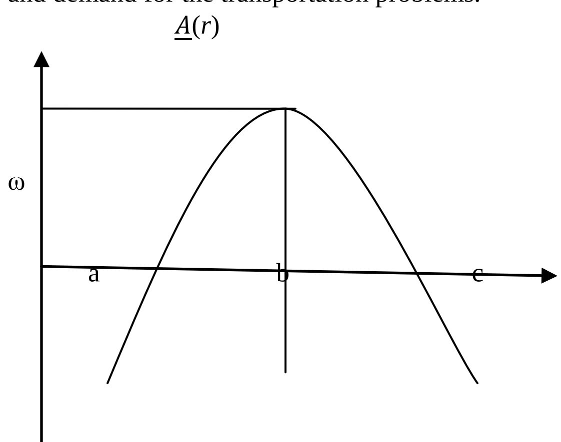


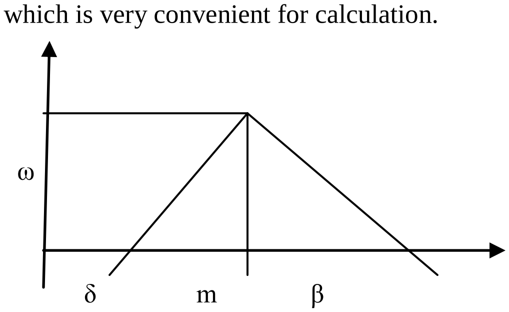
Figure 1: an arbitrary Fuzzy number ($0 \le \omega \le 1$)

According to the above-mentioned definition of a triangular fuzzy number,

let $\tilde{A}$= ($\underline{A}(r)$,$\bar{A}(r)$), ($0 \le r \le 1$) be a fuzzy number, then the value $M(\tilde{A})$, is assigned to $\tilde{A}$ is calculated as follows：

$$M_0^{Tri}(\tilde{A}) = \frac{1}{2}\int_0^1 \{\underline{A}(r) + \ \ \bar{A}(r)\} \ \mathrm{dr} = \frac{1}{4}[2a_2 + a_1 + a_3]$$

which is very convenient for calculation.



A triangular fuzzy number

Definition let $\tilde{A}$= (a_1,a_2, a_3) and $\tilde{B}$ = (b_1,b_2,b_3) be two triangular fuzzy numbers. Then

(1). $\tilde{A} \oplus \tilde{B}$ = (a_1,a_2, a_3) $\oplus$ (b_1,b_2,b_3) = (a_1+b_1,a_2+b_2,a_3+b_3)

(2). $\tilde{A} \ \ominus \ \tilde{B}$ = (a_1,a_2, a_3) $\ominus$ (b_1,b_2,b_3) = (a_1+b_1,a_2-b_2,a_3+b_3)

(3). $\tilde{A} \otimes \tilde{B}$ = (a_1,a_2, a_3) $\otimes$ (b_1,b_2,b_3) = (s_1,s_2,s_3)

where

s1 = minimum{a_1b_1, a_1b_3, a_3b_1,a_3b_3},

s2 = mn

s3 = maximum{ a_1b_1, a_1b_3, a_3b_1,a_3b_3},

3. A new Algorithm for solving fuzzy transportation Problem

Now, we introduce a new method for solving a fuzzy transportation problem where the transportation cost, supply and demands are fuzzy numbers. The fuzzy numbers in this problem is triangular. The optimal solution for the fuzzy transportation problem can be obtained as a crisp or fuzzy form.

Step 1. Calculate the values M(.) for each fuzzy data, the transportation costs $\tilde{c}_{ij}$ supply $\tilde{a}_i$ and demand $\tilde{b}_j$ quantities which are fuzzy quantities.

Step 2. By replacing $M(\tilde{c}_{ij})$, $M(\tilde{a}_i)$ and $M(\tilde{b}_j)$ which are crisp quantities instead of the $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ quantities which are fuzzy quantities, define a new crisp transportation problem.

Step 3. Solve the new crisp transportation problem, by usual method , and obtain the crisp optimal solution of the problem.

Note Any solution of this transportation problem will contain exactly (m+n-1) basic feasible solutions. We note that the optimal solution X_{ij} is to be some positive integer or zero, but the optimal solution X_{ij} for a crisp transportation problem may be integer or non integer, because the RHS of the problem are the measure of fuzzy numbers which are real numbers. If you accept the crisp solution then stop. The optimal solution is in your hand. If you want fuzzy form of solution, go to the next step.

Step 4. Determine the locations of non zero basic feasible solutions in transportation tableau. The basis is rooted spanning tree, that is there must be at least one basic cell in each row and in each column of the transportation tableau. Also, the basis, must be a tree, that is , the (m+n-1) basic cells should not contain a cycle. Therefore, there exist some rows and columns which have only one basic cell. By starting from these cells, calculate the fuzzy basic solutions, continue until obtain (m+n-1) basic solutions.

4.Numerical example

The following example may be helpful to clarify the proposed method:

Example: Consider the following fuzzy transportation problem .All the data in this problem are triangular fuzzy numbers. We want to solve it by our method, and then we will compare the results.

A company has two factories O_1, O_2 and two retail stores D_1, D_2. The production quantities per month at O_1, O_2 are (150, 201, 246) and (50, 99, 154) tons respectively. The demands per month for D_1 and D_2 are (100,150,200) and (100,150,200) tons respectively. The transportation cost per ton $\tilde{c}_{ij}$, i =1, 2; j=1, 2 are $\tilde{c}_{11}$ = (15,19,29), $\tilde{c}_{12}$= (22,31,34), $\tilde{c}_{21}$= (8,10,12) and $\tilde{c}_{22}$= (30,39,54).

Solution: Transportation table of the given fuzzy transportation problem is

Table 4.1 Fuzzy Transportation Problem

	D₁	**D₂**	**Supply**
O₁	(15,19,29)	(22,31,34)	(150,201,246)
O₂	(8,10,12)	(30,39,54)	(50,99,154)
Demand	(100,150,200)	(100,150,200)	(200,300,400)

According to the above-mentioned definition of a triangular fuzzy number,

let $\tilde{A}$= ($\underline{A}(r)$,$\bar{A}(r)$), $(0 \leq r \leq 1)$ be a fuzzy number, then the value $M(\tilde{A})$, is assigned to $\tilde{A}$ is calculated as follows：

$$M_0^{Tri}(\widetilde{A}) = \frac{1}{2}\int_0^1 \{\underline{A}(r) + \bar{A}(r)\}\ dr = \frac{1}{4}[2a_2 + a_1 + a_3]$$

which is very convenient for calculation.

Since $M_o^{Tri}(\tilde{S}) = M_o^{Tri}(\widetilde{D})$, the given problem is a balanced one.

Now, by using our method we change the fuzzy transportation problem into a crisp transportation problem. So, we have the following reduced fuzzy transportation problem:

	D₁	**D₂**	**Supply**
O₁	20.5	29.5	199.5
O₂	10	40.5	100.5
Demand	150	150	300

Table 4.2

As shown in the table 4.2, the result of defuzzification of the fuzzy numbers obtaining the measures which are not all integer. So, existing of a non integer value in a transportation problem follows this fact that the solution of the crisp transportation problem is not integer. We note that the solution in a usual transportation problem is integer, because its matrix is an unimodular matrix

If we solve the new problem, we obtain the solutions as follows:

X11 = 49.5, *X*12 = 150, *X*21 = 100.5

	D₁		**D₂**		**Supply**
O₁	20.5	49.2	29.5	150	199.5
O₂	10	100.5	40.5		100.5
Demand	150		150		300

Table 4.3

Now, we can return to initial problem and obtain the fuzzy solution of the fuzzy transportation problem based on the data of table 4.3.

	D₁	**D₂**	**Supply**
O₁	(350,51,346)	(100,150,200)	(150,201,246)
O₂	(50,99,154)		(50,99,154)
Demand	(100,150,200)	(100,150,200)	(200,300,400)

Table 4.4

where the fuzzy optimal solution for the given fuzzy transportation problem is:

$\tilde{x}_{11}$ = (350, 51,346)

$\tilde{x}_{12}$ = (100,150,200)

$\tilde{x}_{21}$ = (50, 99,154)

The crisp value of the optimum fuzzy transportation cost for the given problem by 6444.5

5. Conclusion

In this paper, the transportation costs are considered as imprecise numbers described by triangular fuzzy numbers which are more realistic and general in nature. In this method, we had used a simple method to solve fuzzy transportation problem by using ranking of fuzzy numbers. This method can be used for all kinds of fuzzy transportation problem, instead of other existing methods. The new method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function.

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