A Study on Anti Fuzzy B Ideals in Homomorphism and Cartesian product on B-Algebras

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Abstract

In this paper, anti fuzzy B-ideals and anti fuzzy B algebras concepts are introduced and proved some results Homomorphism and anti homomorphism functions are satisfied while applying the anti fuzzy B-ideal concept. Anti Fuzzy B-ideal is also applied in Cartesian product.

Keywords:

B-algebras, B-ideals, Fuzzy B-ideals, Anti fuzzy B ideals, Anti fuzzy B-algebras Homomorphism, Anti homomorphism, Cartesian product.

1. Introduction

After the introduction of fuzzy subsets by L.A. Zadeh [10], several researchers explored on the generalization of the notion of fuzzy subset.Y.B. Jun, E.H. Roh, and H.S. Kim [6] introduced a new notion, called a BH- algebra. J. Neggers and H.S. Kim [8] introduced a new notion, called a B-algebra which is related to several classes of algebras of interest such as BCH/BCI/BCK-algebras. J.R. Cho and H.S. Kim [2] discussed further relations between B-algebras and other topics, especially quasi-groups. Y.B. Jun etal [7] fuzzy field (normal) B-algebras and gave a characterization of a fuzzy B-algebras. Sun Shin Ahn and Keumseong Bang [11] gave discussed the fuzzy sub- algebra in B-algebra. C. Yamini and S. Kailasavalli introduced B ideals in B-algebras[12].R. Biswas introduced the concept of Anti fuzzy subgroup of a group [1]. Modifying his idea, in this paper we apply the idea of B-algebras. We introduce the notion of Anti fuzzy B-ideals of B-algebras.

2. Preliminaries

In this section we give some basic definitions and preliminaries of B algebras, B- ideals and fuzzy B- ideals.

Definition 2.1

A B- algebra is a non empty set X with a constant 0 and a binary operation "*" satisfying axioms:

(i) x * x = 0
(ii) x * 0 = x
(iii) (x*y) * z = x * (z * (0*y)), for all x,y,z ∈ X.

For brevity we also call X a B-algebra. In X we can define a binary relation "$\leq$" by x ≤ y if and only if x * y = 0.

Definition 2.2

A non-empty subset I of a B-algebra X is called a subalgebra of X if x * y ∈ I for any x,y ∈ I.

Definition 2.3

Let μ be a fuzzy set in a B-algebra. Then μ is called a fuzzy subalgebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ for all x,y ∈ X.

Definition 2.4

A fuzzy subset μ of a B-algebra is called an anti fuzzy subalgebra of X if

$\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$ for all x,y ∈ X.

Definition 2.5

A nonempty subset I of a B- algebra X is called a B –Ideal of X if it satisfies for x, y, z ∈ X.

(i) 0 ∈ I
(ii) (x * y) ∈ I and (z*x) ∈ I imply (y * z) ∈ I

Theorem: 2.6

If μ is an anti fuzzy subalgebra of a B-algebra X, then $\mu(0) \leq \mu(x)$ for any x∈X.

Proof:

Since x * x =0 for any x∈X

$$\text{Then } \mu(0) = \mu(x * x) \leq \max\{\mu(x), \mu(x)\} = \mu(x)$$

$$\text{Hence } \mu(0) \leq \mu(x).$$

Definition: 2.7

Let μ be a fuzzy set of X. For a fixed t∈[0,1] the set $\mu^t = \{x \in X / \mu(x) \leq t\}$ is called the lower level subset of μ.

Clearly $\mu^t \cup \mu_t = X$ for t ∈ [0,1] if $t_1 < t_2$ then $\mu^{t_1} \subseteq \mu^{t_2}$.

Definition: 2.8

Let X be a B-algebra and μ be a fuzzy subalgebra of X. The subalgebras μ_t, t∈[0,1] and $t \leq \mu(0)$ is called a level subalgebra of μ.

Theorem: 2.9

A fuzzy set μ of a B-algebra X is an anti fuzzy subalgebra if for every t∈[0,1], μ^t is either empty or a subalgebra of X.

Proof:

Assume that μ is an anti fuzzy subalgebra of X and $\mu^t \neq 0$ then for any x,y $\in \mu^t$, we have

$\mu($ x * y $) \leq \max\{\mu(x), \mu(y)\} \leq t$.

Therefore x * y $\in \mu^t$. Hence μ^t is a subalgebra of X.

Conversely for any x,y ∈ X denote by t = $\max\{\mu(x), \mu(y)\}$. Then by assumption μ^t is a subalgebra of X.

Which implies x * y∈ μ^t

Therefore $\mu($ x * y $) \leq t = \max\{\mu(x), \mu(y)\}$

Hence μ is an anti fuzzy subalgebra of X.

Theorem: 2.10

Let μ be a fuzzy set of a B-algebra, X is an anti fuzzy sub algebra such that μ^t is a subalgebra for all t∈[0,1], $t \geq \mu(0)$. Then μ is an anti fuzzy sub algebra of X.

Proof:

Let x,y∈ X and let $\mu(x) = t_1$ and $\mu(y) = t_2$. Then x∈ μ^{t_1} and y∈ μ^{t_2}. Assume that $t_1 \geq t_2$,

Then $\mu^{t_1} \supseteq \mu^{t_2}$ and so y∈ μ^{t_1}. Since μ^{t_1} is a sub algebra of X, we have x * y∈ μ^{t_1}. Thus

$\mu($ x * y $) \leq t_1 = \max\{\mu(x), \mu(y)\}$.

This completes the proof.

Theorem 2.11

Any subalgebra of a B-algebra X can be realized as a level sub algebra of some anti fuzzy subalgebra of X.

Proof:

Let μ be a subalgebra of a given B-algebra X and let μ be a fuzzy set in X defined by

$\mu(x)$= t if x∈A

= 0 if x∉A

Where t∈[0,1] is fixed. It is clear that μ^t=A

Now we prove such defined μ is an anti fuzzy subalgebra of X.

Let x,y∈ X If x,y∈ A then x * y∈ A

Hence $\mu(x) = \mu(y) = \mu(x * y) = t$ and $\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$

If x,y ∉A then $\mu(x)$= $\mu(y)$=0 and $\mu($ x * y $) \leq \max\{\mu(x), \mu(y)\} = 0$.

If at most one of x, y∈A, then at least one of $\mu(x)$ *and* $\mu(y)$ is equal to zero.

Therefore $\max\{\mu(x), \mu(y)\} = 0$ so that $\mu(x * y) \leq 0$.

Which completes the proof.

Theorem: 2.12

Two level subalgebras μ^s, μ^t(s < t) of an anti fuzzy sub algebra are equal if there is no x∈X such that s ≤ $\mu(x)$ < t.

Proof:

Let μ^s=μ^t for some s < t. If there exist x∈X s ≤ $\mu(x)$ < t, then $\mu^t$is a proper subset of μ^s,

Which is a contradiction.

Conversely, Assume that there is no x∈X such that s ≤ $\mu(x)$ < t If x∈ μ^s then $\mu(x) \leq s$

And $\mu(x) \leq t$ Since $\mu(x)$ does not lie between s and t. Thus x ∈ μ^t. Which gives $\mu^s \subseteq \mu^t$

Also $\mu^t \subseteq \mu^s$. Therefore $\mu^s = \mu^t$.

3. B-ideals and antifuzzy B-ideals

Definition 3.1

Let (x, *, 0) be a B-algebra, a fuzzy subset μ in X is called a fuzzy B-Ideal of X if it satisfies the following conditions: for all x, y, z ∈ X.

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(y * z) \geq \min\{\mu(x * y), \mu(z * x)\}$.

Definition 3.2

Let (x, *, 0) be a B-algebra, a fuzzy subset μ in X is called an antifuzzy B-Ideal of X if it satisfies the following conditions for all x, y, z ∈ X.

(i) $\mu(0) \leq \mu(x)$
(ii) $\mu(y * z) \leq \max\{\mu(x * y), \mu(z * x)\}$.

Theorem: 3.3

Every anti fuzzy B-Ideal μ of B-algebra X is order preserving that is y ≤ x then $\mu(y) \leq \mu(x)$ for all x,y ϵ X.

Proof:

Let μ be an anti fuzzy B-Ideal of B-algebra X and let x, y ϵ X such that y ≤ x then y * x = 0

$\mu(y) = \mu(0 * y)$

$\leq \max\{\mu(x * 0), \mu(y * x)\}$

$\leq \max\{\mu(x), \mu(0)\}$

$= \mu(x)$

Hence $\mu(y) \leq \mu(x)$.

Theorem: 3.4

Let μ be a fuzzy B-Ideal of a B-algebra X if μ^c is an anti fuzzy B-Ideal of X

Proof:

Let μ be a fuzzy B-Ideal of X and let x, y, z ϵ X then

(i) $\mu^c(0) = 1 - \mu(0) \leq 1 - \mu(x) = \mu^c(x)$
that is $\mu^c(0) \leq \mu^c(x)$

(ii) $\mu^c(y * z) = 1 - \mu(y * z)$
$\leq 1 - \min\{\mu(x * y), \mu(z * x)\}$
$\leq 1 - \min\{1 - \mu^c(x * y), 1 - \mu^c(z * x)\}$
$= \max\{\mu^c(x * y), \mu^c(z * x)\}$

that is $\mu^c(y * z) \leq \max\{\mu^c(x * y), \mu^c(z * x)\}$

Thus μ^c is an anti fuzzy B-Ideal of X .The converse also can be proved similarly.

4. Homomorphism and Anti Homomorphism of B-algebra

In this section we have discussed about anti fuzzy B-Ideals in B-algebra under homomorphism and some of its properties.

Definition: 4.1

Let $(X,*,0)$ and $(Y, \Delta, 0')$ be B-algebras. A mapping $f : X \rightarrow Y$ is called a homomorphism

if $f(x * y) = f(x) \, \Delta \, f(y)$, for all $x, y \,\epsilon\, X$.

Definition: 4.2

Let $(X, *, 0)$ and $(Y, \Delta, 0')$ be B-algebras. A mapping f: $X \rightarrow Y$ is called an anti homomorphism, if $f(x * y) = f(y) \, \Delta \, f(x)$, for all $x, y \,\epsilon\, X$.

Definition: 4.3

Let $f : X \rightarrow X$ be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as $\mu_f(x) = \mu(f(x))$ for all x in X.

Definition: 4.4

For any homomorphism f: X → Y the set $\{x \in X / f(x) = 0'\}$ is called the Kernal of f, denoted by Ker(f) and the set $\{f(x)/x \in X\}$ is called the image of f denoted by Im(f).

Theorem: 4.5

Let f be an endomorphism of a B – algebra X. If μ is an anti fuzzy B- Ideal of X, then so is μ_f.

Proof:

$\mu_f(0) = \mu(f(0))$

$\leq \mu(f(x))$

$= \mu_f(x)$.

Let x, y, z ϵ X

Then

$\mu_f(y * z) = \mu(f(y * z))$

$= \mu(f(y) * f(z))$

$\leq \max\{\mu(f(x) * f(y)), \mu(f(z) * f(x))\}$

$= \max\{\mu(f(x * y)), \mu(f(z * x))\}$

$= \max\{\mu_f(x * y), \mu_f(z * x)\}$.

Hence μ_f is an anti fuzzy B Ideal of X.

Theorem: 4.6

Let $(X, *, 0)$ and $(Y, \Delta, 0')$ be B-algebras. A mapping f: X → Y is an anti homomorphism of B-algebra. Then Ker(f) is a B-ideal.

Proof:

Let $(x * y) * (z * x) \,\epsilon\, \ker(f)$ & $x \,\epsilon\, \ker(f)$

Then $f((x * y) * (z * x)) = 0'$ and $f(x) = 0'$

$0' = f((x * y) * (z * x))$

$= f(z * x) \, \Delta \, f(x * y)$

$= (f(x) \, \Delta \, f(z)) \, \Delta \, (f(y) \, \Delta \, f(x))$

$= (0' \, \Delta \, f(z)) \, \Delta \, (f(y) \, \Delta \, 0')$

$= f(z) \, \Delta \, f(y)$

$= f(y * z)$

$\Rightarrow y * z \,\epsilon\, \ker(f)$

Hence Ker(f) is a B – ideal .

5. Cartesian Product of fuzzy B- ideals of B-Algebras

In this section, we introduce the concept of Cartesian product of anti fuzzy B – ideals of B algebras.

Definition: 5.1

Let μ and δ be the fuzzy sets in X. The Cartesian product $\mu \times \delta$: X x X → [0 , 1] is defined by $(\mu \times \delta)(x, y) = \min\{\mu(x), \delta(y)\}$ for all x , y ϵ X.

Definition: 5.2

A fuzzy relation R on any set S is a fuzzy subset R: S x S → [0, 1].

Definition: 5.3

Let μ and δ be the anti fuzzy B – ideals in X. The Cartesian product μ x δ : X x X → [0 , 1] is defined by (μ x δ) (x , y) = $\max\{\mu(x), \delta(y)\}$ for all x , y ϵ X.

Definition: 5.4

Let S be a set and μ and δ be fuzzy subsets of S. Then

(i) μ x δ is a fuzzy relation on S,
(ii) $(\mu \text{ x } \delta)_t = \mu_t \times \delta_t$, for all t ∈ [0, 1].

Definition: 5.5

Let S be a set and δ be fuzzy subset of S. The strongest fuzzy relation on S, that is a fuzzy relation on δ is R_δ given by

R_δ(x, y) = min $\{\delta(x), \delta(y)\}$, for all x,y∈S.

Definition: 5.6

For a given fuzzy subset δ of a set S, let R_δ be the strongest fuzzy relation on S. Then for

t∈ [0,1], we have $(R_\delta)_t = \delta_t \times \delta_t$.

Theorem: 5.7

For a given subset δ of a B-algebra X, let R_δ be the strongest fuzzy relation on X. If δ is an anti fuzzy B-ideal of X x X, then $R_\delta(X, X) \geq R_\delta(0{,}0)$ for all x∈X.

Proof:

Given R_δ is an anti fuzzy B ideal.

Since R_δ be the strongest fuzzy relation of X x X, it follows from that

$R_\delta(X, X)$ = max { $\delta(x)$, $\delta(x)$}

≥ max { δ (0), δ (0)}

= $R_\delta(0{,}0)$

Which implies that $R_\delta(X, X) \geq R_\delta(0{,}0)$.

Theorem: 5.8

For a given fuzzy subset δ of a B-algebra X, let R_δ be the strongest fuzzy relation on X. If R_δ is an anti fuzzy B-ideal of X x X then $\delta(X) \geq \delta(0)$ for all x∈X.

Proof :

Since R_δ is an anti fuzzy B-ideal of X x X then

$R_\delta(X, X) \geq R_\delta(0{,}0)$ where (0,0) is the zero element of X x X

But this means that

max $\{\delta(x), \delta(y)\} \geq$ max $\{\delta(0), \delta(0)\}$

which implies that $\delta(x) \geq \delta(0)$.

Theorem: 5.9

If μ and δ are anti fuzzy B-ideals in a B-algebra X, then μ x δ is an anti fuzzy B ideal in X x X.

Proof:

For any (x, y) ϵ X x X we have

(μ x δ) (0, 0) = max{ $\mu(0), \delta(0)$}

≤ max{ $\mu(x), \delta(y)$}

= (μ x δ) (x , y)

Let (x_1,x_2) , (y_1,y_2) , (z_1,z_2) ϵ X x X

(μ x δ) $((y_1,y_2) * (z_1,z_2))$ = (μ x δ) $(y_1 * z_1 , y_2 * z_2)$

= max $\{\mu(y_1 * z_1), \delta(y_2 * z_2)\}$

≤ $max\{\max\{\mu(x_1 * y_1), \mu(z_1 *$ *x1),maxδx2∗y2, δ(z2∗x2)*

= max $\{\max\{\mu(x_1 * y_1), \delta(x_2 *$ *y2,maxμ(z1∗x1), δ(z2∗x2)*

= max $\{\{(\mu \text{ x } \delta)((x_1 * y_1), (x_2 *$ *y2)μ* x *δ(z1∗ x1, z2∗ x2)*

Therefore μ x δ is an anti fuzzy B- ideal in X.

Theorem: 5.10

Let μ and δ be the fuzzy subsets in a B – Algebra X such that μ x δ is an anti fuzzy B-ideal of X x X then for all x∈X,

(i) Either $\mu(0) \leq \mu(x)$ or $\delta(0) \leq \delta(x)$.
(ii) If $\mu(0) \leq \mu(x)$ then either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$.
(iii) If $\delta(0) \leq \delta(x)$ then either $\mu(0) \leq \mu(x)$ (or) $\mu(0) \leq \delta(x)$.
(iv) Either μ or δ is an anti fuzzy B-ideal of X.

Proof:

Let μ x δ be an anti fuzzy B ideal in X x X

Therefore (μ x δ) (0 , 0) ≤ (μ x δ) (x , y) for all (x, y) ϵ X x X

(μ x δ) $((y_1, y_2) * (z_1, z_2))$ ≤$max\{ (\mu \text{ x } \delta)((x_1, x_2) *$ y1,y2, μ x δz1,z2∗x1,x2

For all (x_1, x_2), (y_1, y_2), (z_1, z_2) ϵ X x X

(i) Suppose that $\mu(0) > \mu(x)$ and $\delta(0) > \delta(x)$ for some x , y ϵ X

$(\mu \text{ x } \delta)$ (x, y) = max $\{\mu(x), \delta(y)\}$

$\leq$ max $\{\mu(0), \delta(0)\}$

= $(\mu \text{ x } \delta)$ (0 , 0)

Which is a contradiction.

Therefore $\mu(0) \leq \mu(x)$ or $\delta(0) \leq \delta(x)$ for all $x \in X$.

(ii) Assume that there exist x , y ϵ X such that $\delta(0) > \mu(x)$ and $\delta(0) > \delta(x)$.

Then $(\mu \text{ x } \delta)$ (0 , 0) = max $\{\mu(0), \delta(0)\}$

= $\delta(0)$ and hence

$(\mu \text{ x } \delta)$ (x , y) = max $\{\mu(x), \delta(y)\} < \delta(0)$

= $(\mu \text{ x } \delta)$ (0 , 0)

Which is a contradiction

Hence if $\mu(0) \leq \mu(x)$ for all $x \in X$ then either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$.

Similarly we can prove that if $\delta(0) \leq \delta(x)$ for all $x \in X$ then either

$\mu(0) \leq \mu(x)$ (or) $\mu(0) \leq \delta(x)$ which yields (iii).

(iii) First we prove that δ is an anti fuzzy B-ideal of X

Since by (i) Either $\mu(0) \leq \mu(x)$ or $\delta(0) \leq \delta(x)$ for all $x \in X$

Assume that $\delta(0) \leq \delta(x)$ for all $x \in X$

Then $\delta(x)$ = max $\{\mu(0), \delta(x)\}$

= $(\mu \text{ x } \delta)$ (0 , x)

$\delta(y * z)$ = max $\{\mu(0), \delta(y * z)\}$

= $(\mu \text{ x } \delta)$ (0 , $y * z$)

= $(\mu \text{ x } \delta)$ ($0 * 0$, $y * z$)

= $(\mu \text{ x } \delta)$ ((0 ,y) $*$ (0 , z))

$\leq$ max $\{(\mu \text{ x } \delta)\big(\,(0, \text{x}) *$

0,y,μx δ 0 , z $*$ 0,x

= max $\{(\mu \text{ x } \delta)(0 * 0, \text{x} * \text{y}),$

μx δ(0$*$ 0, z$*$ x)

= max $\{(\mu \text{ x } \delta)(0, \text{x} * \text{y}),$

μx δ0 , z$*$x

= max $\{\delta(x * y), \delta(z * x)\}$

Hence δ is an anti fuzzy B- ideal of x

Next we will prove that μ is an anti fuzzy B- ideal of X.

Let $\mu(0) \leq \mu(x)$

Since by (ii) Either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$.

Assume that $\delta(0) \leq \mu(x)$ then

$\mu(x)$ = max $\{\mu(0), \delta(x)\}$

= $(\mu \text{ x } \delta)$ (x , 0)

$\mu(y * z)$ = max $\{\mu(y * z), \delta(0)\}$

= $(\mu \text{ x } \delta)$ ((y * z), 0)

= $(\mu \text{ x } \delta)$ ((y ,0) * (z , 0))

$\leq$ max $\{(\mu \text{ x } \delta)\big((\text{x}, 0) * (\text{y}, 0)\big),$

μx δz, 0$*$ x, 0

= max $\{(\mu \text{ x } \delta)(\text{x} * \text{y}, 0 * 0),$

μx δz$*$ x, 0$*$ 0

= max $\{\square(x * y), \square(z * x)\}$

Hence μ is an anti fuzzy B-ideal of X.

Theorem: 5.11

Let δ be a fuzzy subset in a B-algebra X and R_δ be the strongest fuzzy relation on X. Then δ is an anti fuzzy B-ideal of X if and only if R_δ is an anti fuzzy B-ideal of X x X.

Proof:

Suppose that δ is an anti fuzzy B – ideal of X.

Then

$R_\delta(0, 0)$ = max $\{\delta(0), \delta(0)\}$

$\leq$ max $\{\delta(x), \delta(y)\}$

= R_δ(x, y), for all (x, y) $\in$ X x X.

For any (x_1, x_2), (y_1, y_2), (z_1, z_2) ϵ X x X.

$R_\delta(y_1 * z_1, y_2 * z_2)$ = max$\{\delta(y_1 * z_1), \delta(y_2 * z_2)\}$

$\leq$ max {max $\{\delta(x_1 * y_1), \delta(z_1 * x_1)\}$, max$\{\{\delta(x_2 * y_2), \delta(z_2 * x_2)\}\}$

= max {max $\{\delta(x_1 * y_1), \delta(x_2 * y_2)\}$, max$\{\delta(z_1 * x_1), \delta(z_2 * x_2)\}$

= max $\{R_\delta(x_1 * y_1, x_2 * y_2), R_\delta(z_1 * x_1, z_2 * x_2)\}$

Hence R_δ is an anti fuzzy B – ideal of X x X.

Conversely, suppose that R_δ is an anti fuzzy B – ideal of X x X, by theorem (5.8), $\delta(0) \leq \delta(x)$ for all $x \in X$.

Now,

Let (x_1, x_2), (y_1, y_2), (z_1, z_2) ϵ X x X.

Then,

max $\{\delta(y_1 * z_1), \delta(y_2 * z_2)\}$ = $R_\delta(y_1 * z_1, y_2 * z_2)$

$\leq$ max $\{R_\delta((x_1, x_2) * (y_1, y_2)), R_\delta((z_1, z_2) * (x_1, x_2))\}$

$= \max\{R_\delta((x_1 * y_1), (x_2 * y_2)), R_\delta((z_1 * x_1), (z_2 * x_2))\}$

$= \max\{\max\{\delta(x_1 * y_1), \delta(x_2 * y_2)\}, \max\{\{\delta(z_1 * x_1), \delta(z_2 * x_2)\}\}$

In particular if we take $x_2 = y_2 = z_2 = 0$, then

$\delta(y_1 * z_1) \le \max\{\delta(x_1 * y_1), \delta(z_1 * x_1)\}$

This proves δ is an anti fuzzy B – ideal of X.

Theorem: 5.12

Let μ *and* δ be a fuzzy subsets of a B-algebra X such that $\mu \times \delta$ is an anti fuzzy B-ideal of X x X. Then μ *or* δ is an anti fuzzy B-ideal of X.

Proof:

(i) By theorem (5.10) (i), without loss of generality we assume that $\mu(x) \ge \mu(0)$ for all x∈X. From the theorem (5.10) (iii) it follows that for all x∈X. either $\delta(0) \le \mu(x)$ (or) $\delta(0) \le \delta(x)$.
If $\delta(0) \le \mu(x)$ for all x∈X
Then $(\mu \times \delta)(0, x) = \max\{\delta(0), \mu(x)\}$
$= \mu(x)$

Let $(x, y) \in X \times X$

Since $\mu \times \delta$ is an anti fuzzy B-ideal of X,

By the theorem 5.9, we get $(\mu \times \delta)(0, 0) \le (\mu \times \delta)(x, y)$

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ using B-ideal

$(\mu \times \delta)(y_1 * z_1, y_2 * z_2) = \max\{\mu(y_1 * z_1), \delta(y2*z2)$

$\le max\{\max\{\mu(x_1 * y_1), \mu(z_1 * x1\delta\ x2*y2, \delta(z2*x2)$

$= max\{\max\{\mu(x_1 * y_1), \delta(x_2 * y2\ \mu(z1*x1), \delta(z2*x2)$

$= \max\{\{(\mu \times \delta)(x_1 * y_1), (x_2 * y2\ (\mu \times \delta)(z1*x1), (z2*x2)$

In particular we take $x_1 = y_1 = z_1 = 0$, then

$\delta(y_2 * z_2) = (\mu \times \delta)(0, , y_2 * z_2)$

$= \max\{\{(\mu \times \delta)(0, x_2 * y2 (\mu \times \delta)(0, z2*x2)$

$\le max\{\max\{\mu(0), \delta(x_2 * y2, \mu(0), \delta(z2*x2)$

$= \max\{\delta(x_2 * y_2), \delta(z_2 * x_2)\}$

This proves that δ is an anti fuzzy B-ideal of X.

The second part is similar.

This completes the proof.

6. Conclusion:

In this article we have discussed Anti fuzzy B-ideals, Anti fuzzy B-algebras, Homomorphism, anti homomorphism and Cartesian product of Anti fuzzy B-ideal of B-algebras.

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