Procedure for Solving Unbalanced Fuzzy Transportation Problem for Maximizing the Profit

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Abstract: *A new method for maximizing profit in unbalanced transportation problems in fuzzy set theory is proposed. With the help of numerical example, the proposed method is illustrated; we use Fuzzy transportation to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers.*

Key words: *Fuzzy numbers, trapezoidal fuzzy numbers, fuzzy Vogel's approximation method.*

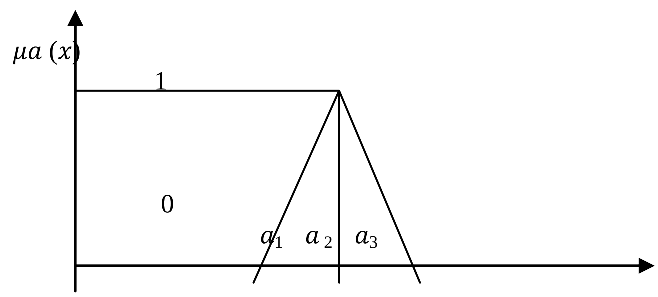
1. Introduction

Transportation models provide a powerful framework to meet the challenge of how to supply the commodities to the customers in more efficient ways. They ensure the efficient movement and timely availability of raw materials and finished goods. In 1941, Hitchcock [2] originally developed the basic transportation problem. In 1953, Charnes et al[5] developed the stepping stone method which provided an alternative way of determining the simplex method information. In 1963, Dantzig [6] used the simplex method to the transportation problems as the primal simplex transportation method. Till date, several researchers studied extensively to solve cost minimizing transportation problem in various ways. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh[17] may represent this data. So, fuzzy decision making method is needed here.Zimmermann[3] showed that solutions obtained by fuzzy linear programming are always efficient. In this paper, we propose a new method where the objective is to maximize the profit by converting the maximization problem into a minimization problem by new method for finding an optimal solution for unbalanced transportation problems

2. Basic definitions

1. **Fuzzy set:** A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. $A = \{(x, \mu_A(x); x \in X\}$, Here $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \varepsilon X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2. **Definition** The fuzzy number $a = a_1, a_{2,3}$ is a triangular fuzzy numbers, denoted by a_1,a_2,a_3 its membership function μ_a is given below the figure.



F(R) represents the set all trapezoidal fuzzy numbers
It R be any ranking function , then , $R(a) = (a_1+a_2+a_3)/3$
Let $a = [a_1,a_2,a_3,]$ and $b = [b_1,b_2,b_3]$ be two triangular fuzzy numbers
then the arithmetic operations on a and b as follows.

Addition: $a + b = (a_1+b_1,\ a_2+b_2,\ {}_3+b_3)$

Subtraction: $a - b = (a_1-b_1,\ a_2-b_2,\ a_3-b_3)$

Multiplication:

$a.\ b = \frac{a_1}{3}(b_1+b_2+b_3),\ \frac{a_2}{3}(b_1+b_2+b_3),\ \frac{a_3}{3}(b_1+b_2+b_3)$
$if\ R(a) > 0$

$a \,.\, b = \frac{a_3}{3}(b_1+b_2+b_3)\,,\frac{a_2}{3}(b_1+b_2+b_3)\,,\frac{a_1}{3}(b_1+b_2+b_3)$ if $R\,(a) < 0$

3.Fuzzy balanced and unbalanced Transportation problem

The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

$$minimize \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} * x_{ij}$$

Subject to $\sum_{j=1}^{q} x_{ij} = \tilde{a}_i$, i = 1,2,3,………,p

$\sum_{i=1}^{p} x_{ij} = b_j$, j = 1,2,3,………,q

$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$

X_{ij} is a non- negative trapezoidal fuzzy number ,

Where p = total number of sources

Q = total number of destinations

a_i = the fuzzy availability of the product at i^{th} source

b_j = the fuzzy demand of the product at j^{th} destination

c_{ij} = the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination

x_{ij} = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost. $\sum_{i=1}^{p} a_i$=1total fuzzy availability of the product,

$\sum_{j=1}^{q} b_j$=1total fuzzy demand of the product

$\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} * x_{ij}$=total fuzzy transportation cost .

If $\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.Consider transportation with m fuzzy origin s (rows) and n fuzzy destinations (Columns) Let $C_{ij}=[C_{ij}^{(1)},C_{ij}^{(2)},C_{ij}^{(3)}]$ be the cost of transporting one unit of the product from i^{th} fuzzy origin to j^{th} fuzzy destination $a_i=[a_i^{(1)},a_i^{(2)},a_i^{(3)}]$ be the quantity of commodity available at fuzzy origin i $b_j=[b_j^{(1)},b_j^{(2)},b_j^{(3)}]$ be the quantity of commodity requirement at fuzzy destination j. $X_{ij}=[X_{ij}^{1},X_{ij}^{2},X_{ij}^{3}]$ is quantity transported from i^{th} fuzzy origin to j^{th} fuzzy destination. An unbalanced transportation problem is converted into a balanced transportation problem by introducing a dummy origin or dummy destinations which will provide for the excess availability or the requirement the cost of transporting a unit from this dummy origin (or dummy destination) to any place is taken to be zero. After converting the unbalanced problem into a balanced problem, we adopt the usual procedure for solving a balanced transportation problem.

Numerical Example:

Let us solve the unbalanced fuzzy transportation problem for maximizing the profit,

	$FD1$	$FD2$	$FD3$	$FD4$	Fuzzy Available
$F01$	3	3	3	1	2.3
$F02$	9.6	8	5	4	6
$FO3$	7	5	5	8	4.3
Fuzzy Requirement	4	2.6	4	4.6	

	$FD1$	$FD2$	$FD3$	$FD4$	Fuzzy Available
$F01$	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,3)	(0,2,5)
$F02$	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(1,6,11)
$FO3$	(2,7,12)	(0,5,10)	(0,5,10)	(4,8,12)	(1,4,8)
Fuzzy Requirement	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	

After ranking:

Since the given problem is a maximization type, first convert into this into a minimization problem by subtracting the cost elements (entries or c_{ij}) from the highest cost element (c_{ij} = 9.6)in the given transportation problem.

	$FD1$	$FD2$	$FD3$	$FD4$	Fuzzy Available
$F01$	6.6	6.6	6.6	8.6	2.3
$F02$	0	1.6	4.6	5.6	6
$FO3$	2.6	4.6	4.6	1.6	4.3
Fuzzy Requirement	4	2.6	4	4.6	

This problem is unbalanced fuzzy transportation problem then the problem convert to balanced fuzzy transportation problem defined as follows.

	$FD1$	$FD2$	$FD3$	$FD4$	Fuzzy Available
$F01$	6.6	6.6	6.6	8.6	2.3
$F02$	0	1.6	4.6	5.6	6
$FO3$	2.6	4.6	4.6	1.6	4.3
Fuzzy Requirement	4	2.6	4	4.6	

The modified minimization problem is unbalanced. To make it balance we introduce a dummy destination FO4 with demand 2.6 units with zero costs c_{ij}. Hence the balanced minimization transportation problem becomes

	$FD1$	$FD2$	$FD3$	$FD4$	Fuzzy Available
$F01$	6.6	6.6	6.6	8.6	2.3
$F02$	0	1.6	4.6	5.6	6
$FO3$	2.6	4.6	4.6	1.6	4.3
FO4	0	0	0	0	2.6
Fuzzy Requirement	4	2.6	4	4.6	

Since $\sum a_i = \sum b_j$ there exist a basic feasible solution to this problem and is displayed in the following table by using VAM.

	$FD1$	$FD2$	$FD3$	$FD4$	Fuzzy Available
$F01$	6.6	6.6	6.6	8.6	2.3
$F02$	0 $\boxed{2.3}$	1.6	4.6	5.6	6
$FO3$	2.6 $\boxed{1.7}$	4.6 $\boxed{2.6}$	4.6 $\boxed{1.7}$	1.6	4.3
FO4	0	0	0 $\boxed{2.3}$	0 $\boxed{2}$	2.6
Fuzzy Requirement	4	2.6	4	4.6 $\boxed{2.6}$	

The above table satisfies the rim conditions with (m+n-1) non negative allocations at independent positions.
The optimum profit = 40.94

Conclusion

We have thus obtained an optimal solution for a fuzzy unbalanced transportation problem using triangular fuzzy numbers for maximizing the profit. A new computational procedure to find the optimal solution is also discussed. The new arithmetic operation triangular fuzzy numbers are employed to get the fuzzy optimal solution. The same method can be used for solving various types of fuzzy problems.

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